Take $0<h<1$, and compare (4) with $1+|x|+|x|^2+\ldots$.

Hence $$|R'|<\frac{1}{1-|x|}, \text{ if } |x|<1.$$

Hence $$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots+(-1)^{r-1}\frac{x^r}{r!}R'_0,$$

where R'_0 is the limit of R', and is therefore $<1/(1-|x|)$.

[The existence of R'_0 is easily established: if $x<0$, R' increases as h diminishes: if $x>0$, R' is the difference of the sums of two series, which sums remain finite and increase as h diminishes.]

6°. The expression $$\underset{n\to\infty}{\text{Lt}}\left(1+\frac{1}{n}\right)^{nx}=\underset{n\to\infty}{\text{Lt}}\, e^{nx\log(1+1/n)}$$
$$=\underset{n\to\infty}{\text{Lt}}\, e^{nx(1/n-1/2n^2R')}=\underset{n\to\infty}{\text{Lt}}\, e^{x(1-1/2nR')},$$

where $|R'|<1\Big/\left(1-\frac{1}{n}\right)$; so that $|R'|<2$ for $n>2$.

Hence $$\underset{n\to\infty}{\text{Lt}}\left(1+\frac{1}{n}\right)^{nx}=e^x.$$

H. Bryon Heywood.

Bedford College, W.

373. **[I. 17.]** *A few cases of factors for a sum of two squares.*

(i) $$x^{12}+(4x^2y^4+2y^6)^2$$
$$=(x^6+4x^5y+8x^4y^2+10x^3y^3+8x^2y^4+4xy^5+2y^6)$$
$$\times(x^6-4x^5y+8x^4y^2-10x^3y^3+8x^2y^4-4xy^5+2y^6).$$

If $x=1$ and $y=10$, we get
$$(2040000)^2+1=(2490841)\times(1670761).$$

(ii) $$(x^6-8x^4y^2+8x^2y^4)^2+(4x^2y^4-2y^6)^2$$
$$=(x^6+4x^5y-10x^3y^3+4xy^5+2y^6)$$
$$\times(x^6-4x^5y+10x^3y^3-4xy^5+2y^6).$$

(iii) $$(8x^6+36x^4y^2)^2+(27x^2y^4+81y^6)^2$$
$$=(8x^6+24x^5y+72x^4y^2+144x^3y^3+189x^2y^4+162xy^5+81y^6)$$
$$\times(8x^6-24x^5y+72x^4y^2-144x^3y^3+189x^2y^4-162xy^5+81y^6).$$

If $x=10$ and $y=1$, this gives
$$(8360000)^2+(2781)^2=11284601\times 6193361.$$

(iv) $$(2x^6-150x^4y^2)^2+(18x^2y^4+y^6)^2$$
$$=(2x^6+36x^5y+174x^4y^2+110x^3y^3+36x^2y^4+6xy^5+y^6)$$
$$\times(2x^6-36x^5y+174x^4y^2-110x^3y^3+36x^2y^4-6xy^5+y^6).$$

(v) $$(x^6+8x^4y^2)^2+(18x^2y^4+81y^6)^2$$
$$=(x^6+2x^5y+12x^4y^2+21x^3y^3+36x^2y^4+54xy^5+81y^6)$$
$$\times(x^6-2x^5y+12x^4y^2-21x^3y^3+36x^2y^4-54xy^5+81y^6).$$

A sum of three squares which has factors:
$$(2x^4)^2+(x^2y^2)^2+(2y^4)^2$$
$$=(2x^4+2x^3y+x^2y^2+2xy^3+2y^4)$$
$$\times(2x^4-2x^3y+x^2y^2-2xy^3+2y^4).$$

G. Osborn.

374. **[A. 3. b.]** *Any symmetric function of the roots of an equation is a function of the coefficients.*

(The following direct proof is shorter than the usual one depending on Newton's Theorem. The process occurs in an example, Burnside and Panton, p. 325, 3rd ed.)

Let the n roots be $\alpha, \beta, \ldots \nu$, and let the assigned function be $\Sigma \alpha^a \beta^b \ldots \lambda^l$, where each term involves p different roots, and the highest index of any one root is q.

If $q=1$, then Σ is equal to a coefficient; if $p=n$, then Σ is the product of the last coefficient and a symmetric function with a lower q. So the theorem will be proved if we can express any symmetric function in terms of others with either a lower q, or the same q and a higher p; for with each of the latter we can continue until $p=n$, and so obtain a lower q in every sum; and then we can repeat the process until $q=1$.

Now consider the product $\Sigma \alpha\beta \ldots \lambda . \Sigma \alpha^{a-1}\beta^{b-1} \ldots \lambda^{l-1}$, of which the first factor has $q=1$, and the second has a lower q than the assigned function. This product is equal to the sum of a set of symmetric functions, of which none has a higher q than the assigned function, and each has a higher p except one, and that one is the assigned function itself, which can therefore be expressed in the way described above. This proves the theorem, and Newton's as a particular case. H. P. HUDSON.

375. [I. 2. b.] May I make the following remarks apropos of Mr. Lupton's article "Furor Arithmeticus" (*Math. Gaz.*, May 1910, pp. 273 *et seq.*)?

(i) The value given for 40! is incorrect; this will be readily seen from the fact that the number is not a multiple of nine. M[lle] and MM. Chanzy, of Nancy, have found

$$40! = 815\,915\,283\,247\,897\,734\,345\,611\,269\,596 \ldots.$$

(ii) The largest known primes,

$$5 . 2^{75} + 1 = 188\,894\,659\,314\,785\,808\,547\,841,$$

which is a factor of Fermat's number F_{73}, or $2^{2^{73}}+1$ (*v.* Dr. J. C. Morehead's article in the *Bull. of the Amer. Math. Soc.* 1906, translated into French in *Sphinx Oedipe*, April 1911).

According to Mr. Powers, of Denver, Colorado, the number $2^{83}-1$ is a prime. This number has 27 digits (*Amer. Math. Monthly*, Nov. 1911).

Edouard Lucas proved that $2^{127}-1$ is a prime. This number has 39 digits (*Bull. du prince Boncompagni*, Rome, 1877).

(iii) I have a complete "historique" of π, giving 707 decimal places. If any reader of the *Gazette* would care to see them they are at his disposal.

A. GÉRARDIN.

376. [I. 2.] *The Introduction to the Idea of a Negative Number.*

In the review of Prof. Tannery's book in your December issue, it is mentioned that in leading up to the idea of a negative number, the author confines himself to concepts arising out of the idea of number, using the series:

$$\ldots -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \ldots.$$

It is difficult to believe that this series of numbers is intelligible to a *beginner*, unless made real and concrete by some such device as your reviewer suggests. I have frequently tested boys who have been taught the manipulation of negative numbers by the method suggested above (without the looking-glass device suggested by your reviewer): I have given the boys the series of numbers in any order and told them to arrange them in the right order in one series. The result is nearly always:

$$0 \quad -1 \quad -2 \quad -3 \quad 1 \quad 2 \quad 3.$$

This result is given by boys who are neither backward nor dull. Even the few who can arrange the numbers in the right order (and these are not always those who can manipulate negative numbers correctly) say that they do not understand the series, and that it is no help to them.

A boy who has worked at algebra in which $x-y$ is intelligible only when x is greater than y has no idea of negative numbers at all; his ideas of