THE THERMOPHONE AS A PRECISION SOURCE OF SOUND.

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THE acoustic effect accompanying the passage of an alternating current through a thin conductor has been known for some time, but as far as we are aware, no use has been made of the principle involved for the production of a precision source of sound energy, or standard phone. In 1898 F. Braun[1] discovered that acoustic effects could be produced by passing alternating currents through a bolometer in which the usual direct current was also maintained. An article by Weinberg[2] describes the old experiments of Braun, and also some experiments of Weinberg, in which acoustic phenomena were observed with other electrically heated conductors, rheostats, etc., through which large alternating currents were passed. A more recent application of the same principle is described by de Lange[3] in his article on the thermophone.

The writers have found that the thermophone together with a suitable supply of alternating current can be used very conveniently as a precision source of sound energy. On account of the fact that the published material on this electrical-acoustic effect is largely of a qualitative character it has been necessary to work out a quantitative theory; and it is the purpose of this paper to give the theory and show how the instrument can be adapted to acoustic measurements.

When alternating current is passed through a thin conductor, periodic heating takes place in the conductor following the variations in current strength. This periodic heating sets up temperature waves which are propagated into the surrounding medium; the amplitude of the temperature waves falling off very rapidly as the distance from the conductor increases. On account of the rapid attenuation of these temperature waves, their net effect is to produce a periodic rise in temperature in a limited portion of the medium near the conductor, and the thermal expansion and contraction of this layer of the medium determines the amplitude of the resulting sound waves. To secure appreciable amplitudes with currents of ordinary magnitude it is essential that the con-

[1] Ann. der Physik. 65, 1898, p. 358.
[2] Elektrot. Zeit. 28, 1907, p. 944. See also A. Koepsel, Elektrot. Zeit. 28, 1907, p. 1095.
[3] Proc. Royal Soc. 91A, 1915, p. 239.

ductor be very thin; its heat capacity must be small, and it must be able to conduct at once to its surface the heat produced in its interior, in order to follow the temperature changes produced by a rapidly varying current.

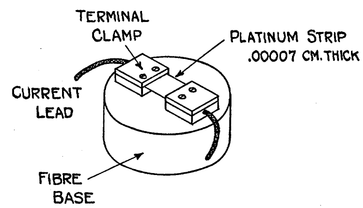

Fig. 1.

Simple Thermophone.

A simple form of instrument which we have used is shown in Fig. 1. There are two ways in which the strip may be supplied with electrical energy in order to produce sound waves, (*a*) with pure alternating current and (*b*) with alternating and direct current superimposed. If an alternating current $I \sin pt$ is supplied, the heating effect is proportional to

$$RI^2 \sin^2 pt = \frac{RI^2}{2}(1 - \cos 2pt), \tag{1}$$

so that the acoustic frequency is double the frequency of the applied alternating current. If it is desired to make the acoustic wave follow the alternating current wave, without introducing the double frequency effect, resort must be had to a superimposed direct current whose value is several times as large as the maximum value of the alternating current. If a direct current I_0 and an alternating current $I' \sin pt$ are used to heat the strip, the heating effect is proportional to

$$\begin{aligned} R(I_0 + I' \sin pt)^2 &= RI_0^2 + 2RI_0I' \sin pt + RI'^2 \sin^2 pt \\ &= R\left(I_0^2 + \frac{I'^2}{2}\right) + 2RI_0I' \sin pt - \frac{RI'^2}{2} \cos 2pt \end{aligned} \tag{2}$$

from which it is evident that the double frequency term can be made negligible by suitable choice of I_0 and I'.

When pure alternating current is used, the mean temperature of the strip is determined by the term $\frac{1}{2}RI^2$; when direct current is used with the alternating current, the mean temperature is determined by the term RI_0^2. The mean temperature of the conductor is one of the factors which sets a limit on the maximum amount of electrical energy used and hence on the maximum amount of acoustic energy that can be obtained. If only a small quantity of alternating current energy of suitable frequency is available, it is clear, from a comparison of equations (2) and (1) that more acoustic effect will be realized if direct current energy is added up to the limit that the strip will bear; for example, if I_0^2 is as large as I'^2, the product term in (2) is four times as large as the second term in (1).

Suppose now that an indefinite quantity of alternating current energy of any frequency is at hand; we desire to find the most effective way to actuate the element. Equating the terms in (1) and (2) which are proportional to the (limiting) mean temperature in each case

$$\left[\frac{RI^2}{2}\right]_{\max} = \left[R\left(I_0^2 + \frac{I'^2}{2}\right)\right]_{\max} \tag{3}$$

we can compute the maximum amplitude $2RI_0I'$ of the product term in (2) and compare this with the amplitude $RI^2/2$ of the periodic term in (1). The maximum value of the product $2RI_0I'$, consistent with condition (3) is $RI^2/\sqrt{2}$ and implies the relations

$$I = \sqrt{2}\, I' = 2I_0.$$

The amplitude $RI^2/\sqrt{2}$ in the second case is only slightly larger than the amplitude $RI^2/2$ which we should have according to (1); and in the second case there is the double frequency term of amplitude $RI^2/4$ which in most cases would be inconveniently large.

The conclusion from these calculations is that for sounding a pure tone of a given frequency it is better to actuate the strip wholly by alternating current of half that frequency. However, if it is desired to make the sound waves reproduce the electrical waves in both frequency and form, it is necessary to use in addition a direct current whose relative value is large. In this case the thermophone element is worked somewhat below maximum efficiency for the sake of minimizing the double-frequency effect.

Using the first method of excitation, it is necessary, if a pure tone is desired, that the alternating current used be a pure sine wave, absolutely free from harmonics. In order to show the acoustic effect of harmonics in the alternating current supply, consider an exciting current of the form

$$\sum_{k=1}^{n} a_k \sin kpt.$$

The heating effect produced is proportional to

$$\left(\sum_{k=1}^{n} a_k \sin kpt\right)^2 = \sum_{k=1}^{n} a_k^2 \sin^2 kpt + \sum_{j=1}^{n-1}\sum_{\substack{k=2\\ j\neq k}}^{n} a_j a_k \sin jpt \sin kpt$$

$$= \sum_{k=1}^{n} \frac{a_k^2}{2} - \sum_{k=1}^{n} \frac{a_k^2}{2} \cos 2kpt + \sum_{j=1}^{n-1}\sum_{\substack{k=2\\ j\neq k}}^{n} \frac{a_j a_k}{2} \cos (j-k)pt$$

$$- \sum_{j=1}^{n-1}\sum_{\substack{k=2\\ j\neq k}}^{n} \frac{a_j a_k}{2} \cos (j+k)pt.$$

which shows that two series of combination-tones result in addition to the series of tones whose frequencies are double those of the applied fundamental and harmonics. One particular case is of practical importance: the case in which the alternating current wave consists of a fundamental and an appreciable second harmonic. In this case, besides the tones of double and quadruple frequency there are combination tones of *single* and triple frequency, a paradoxical result that is very easily verified by experiment. The importance of a pure alternating current supply is clear from the considerations given.

The Periodic Temperature Change in a Thin Flat Conductor Supplied with Alternating Current.

Consider first the case of a strip supplied with both direct and alternating current. Equating the rate of production of heat by the electric current to the rate of transfer of heat to the surrounding medium, plus the rate of storage of heat in the strip, the fundamental equation may be written:

$$0.24(I_0 + I' \sin pt)^2 R = 2a\beta T + a\gamma \frac{dT}{dt}, \tag{4}$$

in which the unit is the calorie per second, and the constants are chosen as follows:

I_0 = direct current in amperes.
I' = maximum value of A.C. in amperes.
p = $2\pi f$; f = frequency.
R = instantaneous resistance of the strip.
T = temperature of strip above surroundings.
a = area of one side of strip.
β = the rate of loss of heat per unit area of the strip (due to conduction *and* radiation) per unit rise in temperature of the strip above that of its surroundings; it is equal to the product of the temperature gradient per degree rise, into the conductivity of the medium. It can be determined experimentally, and is a constant if only conduction is considered; if it is desired to take account of radiation a modified value of β for any value of T may be obtained which is sufficiently accurate for the purposes of calculation. The rate of radiation is not great at low temperatures, and only becomes equal to the rate of conduction at about 500° C.
$a\gamma$ = the heat capacity of the strip, γ being equal to the product of the thickness of the strip by the specific heat per unit volume.

The factor is analogous to the mass in vibratory mechanics, and the inductance in alternating current calculations.

The equation for T_0, the mean temperature above surroundings is:

$$0.24\left(I_0^2R + \frac{I'^2R}{2}\right) = 2a\beta T_0. \qquad (4a)$$

Combining equations (4) and (4a) we have the following, which contains only factors which vary with the time:

$$0.24R\left(2I_0I'\sin pt - \frac{I'^2}{2}\cos 2pt\right) = 2a\beta(T - T_0) + a\gamma\frac{dT}{dt}. \qquad (5)$$

In obtaining a solution for $T - T_0$ we shall neglect transient effects, also the double frequency term. The double frequency effect is the principal effect in the case of a pure alternating current supply as given below; but here we simply remark that we can make the double frequency term as small as we please by a suitable choice of the ratio $I_0 : I'$.

The solution of the equation

$$.48RI_0I'\sin pt = 2a\beta(T - T_0) + a\gamma\frac{dT}{dt} \qquad (6)$$

is, neglecting transient effects,

$$T - T_0 = \frac{.48RI_0I'}{a\sqrt{4\beta^2 + \gamma^2p^2}}\sin\left(pt - \tan^{-1}\frac{\gamma p}{2\beta}\right), \qquad (7)$$

which gives the periodic temperature variation of the strip. Note that if i is the effective (measured) value of the A.C., $\sqrt{2}\,i$ must be written in place of I' in (7).

If the strip is supplied with alternating current only, the fundamental equation becomes

$$0.24RI^2\sin^2 pt = .12RI^2(1 - \cos 2pt) = 2a\beta T + a\gamma\frac{dT}{dt}. \qquad (4')$$

The mean temperature in this case is defined by

$$.12RI^2 = 2a\beta T_0 \qquad (4'a)$$

and the differential equation which $T - T_0$ must satisfy is

$$.12RI^2\cos 2pt = 2a\beta(T - T_0) + a\gamma\frac{dT}{dt}. \qquad (5')$$

The solution of this equation is, neglecting transient effects

$$T - T_0 = \frac{.12RI^2}{2a\sqrt{\beta^2 + \gamma^2p^2}}\cos\left(2pt - \tan^{-1}\frac{\gamma p}{2\beta}\right). \qquad (7')$$

Having found the magnitude of the temperature variation in the strip, we go on to calculate the magnitude of the effect in the surrounding medium.

Theory of the Effect in the Medium.

Consider an infinite plane metal plate with a column of gas extending normally from either face of a certain portion of the plate; this is equivalent, mechanically, to the strip conductor if terminal conditions are neglected. If the temperature of the plate is a sine function of the time, temperature waves will be propagated into the atmosphere on either side; and calculation will show that these waves are so heavily damped that they are practically extinguished after one wave-length has been traversed. Within this region there is a rise and fall of temperature of the medium with every cycle, and the resulting expansion and contraction of this narrow film of the medium near the source accounts for the sound vibration produced.

In the derivation of equations (7) and (7′) it has been tacitly assumed that no electrical energy was spent in expanding the strip, as this effect would be relatively very small. It is evident from conditions of symmetry that there is no force on the strip tending to make it vibrate; hence no energy can be used mechanically. In calculating the effect on the medium we shall consider two cases:

1. In which the periodic rise in temperature of the strip is allowed to produce a continuous stream of sound energy, propagated away from the strip as plane waves. It is an easy matter to modify this treatment to fit the case of diverging waves in the open atmosphere.

2. In which the strip is placed in a small cavity for the purpose of producing pressure changes; these pressure changes being used to actuate the ear, or some mechanical phonometer which constitutes one wall of the enclosure.

The reason for giving separate treatment to these two types of action, is that in the first case we can speak of a definite amplitude and particle velocity, and a corresponding propagation of energy; whereas in the second case, amplitude and velocity are indefinite terms, and pressure change is much more readily calculated. It is by virtue of pressure change that the acoustic energy generated makes its effect on the bounding wall, and if the dimensions of the cavity are small compared to the acoustic wave-length, the pressure change produced at the strip is quickly distributed over the whole enclosure.

First Case: Wave Propagation from the Strip.—Assume that the periodic temperature variation in the strip results in the expansion and contraction against constant atmospheric pressure of a certain layer of air next to the source. This implies that the very small pressure changes that do arise at the boundary of the layer (as the result of rapid change in volume) are propagated into the atmosphere with such high velocity that they do not react appreciably on the expansion of the layer. This condition is realized in practice because the velocity of sound in air is so much greater than the velocity of the vibrating boundary which produces the sound.

In treatises on the conduction of heat it is shown that the temperature at any point of the medium distant $\pm x$ from a plane source of temperature, varying periodically as in equation (7), may be expressed as the following function of space and time:

$$T_x' = T'\epsilon^{-\alpha x} \sin (pt \pm \alpha x), \tag{8}$$

in which $\alpha = \sqrt{p/2k}$, p being $2\pi \times$ frequency, and k the "diffusivity" of the medium, or the ratio of the thermal conductivity to the specific heat per unit volume. The value of this constant for air at 0° centigrade, using the specific heat at constant pressure is 0.17 C.G.S. units.

It is necessary to know the effect of the temperature of the medium on k and this can be found by considering separately the conductivity and the specific heat. The former is proportional to the square root of the absolute temperature; the specific heat per unit mass is practically independent of temperature thus making the specific heat per unit volume proportional to the reciprocal of the absolute temperature. Since k is the ratio, we may write

$$k = 0.17 \left(\frac{\theta}{273}\right)^{3/2} \tag{9}$$

where θ denotes the absolute temperature of the medium.

The velocity of propagation of the temperature wave is, from (8)

$$v = \frac{p}{\alpha} = \sqrt{2pk}$$

and the wave-length

$$\lambda = \frac{2\pi}{\alpha} = \sqrt{\frac{8\pi^2 k}{p}}. \tag{10}$$

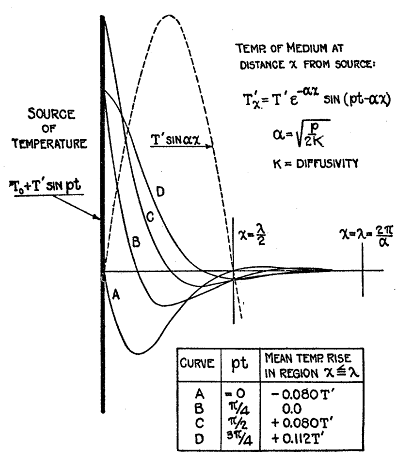

Fig. 2.

If the wave-length is taken as a unit, it is easy to plot the course of T_x' as a function of x for any given time, t, as is shown in Fig. 2. This shows clearly the enormous damping of these waves of acoustic frequency; it also shows that practically all of the expansion effect due to periodic rise in temperature takes place within the region bounded by the plane

$$x = \lambda = \frac{2\pi}{\alpha}.$$

In order to compute the amount of the periodic expansion, we desire to know the mean value of the temperature rise in this region as a function of the time: that is,

$$\frac{T'}{\lambda}\int_0^\lambda \epsilon^{-\alpha x}\sin(pt-\alpha x)dx$$
$$=\frac{T'(1+\epsilon^{-2\pi})}{2\lambda\alpha}(\sin pt-\cos pt)=\frac{T'}{2\sqrt{2\pi}}\sin\left(pt-\frac{\pi}{4}\right) \quad (11)$$

i. e., its maximum value is $.112T'$, and it lags the varying temperature of the strip by the angle $\pi/4$.

If the mean absolute temperature of the air film is θ, the maximum expansion will be,

$$\frac{d\theta}{\theta}=.112\frac{T'}{\theta}\text{ per unit volume,} \quad (12)$$

or per unit length, if expansion is considered to take place in only one direction. The length in question is a wave-length, and this by equations (9) and (10) is, at $\theta°$

$$\lambda=2\pi\sqrt{\frac{2k}{p}}=0.82\sqrt{\frac{\pi}{f}}\left(\frac{\theta}{273}\right)^{3/4}. \quad (13)$$

Multiplying (12) and (13) we obtain for the absolute increase in length due to expansion

$$\xi=\frac{.16T'}{\sqrt{f^{\frac{1}{2}}\theta^{\frac{1}{4}}(273)^{\frac{3}{4}}}}. \quad (14)$$

This may be considered as the maximum "amplitude" of a sound wave leaving the plane $x=\lambda$, if the effect of the expanding and contracting air film on the surrounding air is the same as that of a solid moving piston—assuming also that the amplitude of the sound produced by a moving piston is equal to the amplitude of the motion of the piston itself.

If the thermal conductivity were proportional to the first power of the absolute temperature, instead of to the square root, we should have had, instead of (12)

$$\xi_1=\frac{0.16T'}{\sqrt{f}\,.273}. \quad (14a)$$

The departure of (14*a*) from (14) is not serious, if the temperature of the film is 300° C. or less, as in this case $\theta^{1/4}\cdot(273)^{3/4}$ is less than 330. The air film is always considerably cooler than the strip, so that the strip might have a temperature of (say) 500° without causing more than a 20 per cent. discrepancy between ξ and ξ_1.

In order to have the amplitude of the sound wave in terms of the alternating current supplied to the strip, we make use of equations (7) and (7′) which give the variation in temperature of the strip.

Using (7) and (14a), we have for a strip supplied with direct current I_0 and alternating current of effective value i, the acoustic amplitude

$$\xi = \frac{4.0 \times 10^{-4} R I_0 i}{a\sqrt{f}\sqrt{4\beta^2 + \gamma^2 p^2}} \sin\left(pt - \tan^{-1}\frac{\gamma p}{2\beta} - \frac{\pi}{4}\right). \qquad (15)$$

Using (7′) and (14a) we have for a strip supplied with alternating current only

$$\xi' = \frac{7.0 \times 10^{-5} R i^2}{a\sqrt{f}\sqrt{\beta^2 + \gamma^2 p^2}} \cos\left(2pt - \tan^{-1}\frac{\gamma P}{\beta} - \frac{\pi}{4}\right). \qquad (15')$$

These two equations contain no transient terms; they are solutions for the state of steadily maintained vibrations. The acoustic amplitude ξ' (Equation 15′) is of double the frequency of the applied alternating current.

Using either method of actuating the strip, there is a low critical frequency above which the factor γp (which represents thermal inertia) is so much greater than β (which represents conduction loss, or dissipation) that the latter can be neglected. (This frequency is in the neighborhood of 100 for platinum strip 1 micron thick.) Neglecting β, (15) can be written

$$\xi = \frac{6.4 \times 10^{-5} R I_0 i}{a\gamma f^{3/2}} \sin\left(pt - \frac{3\pi}{4}\right), \qquad (15a)$$

and instead of (15′) we have

$$\xi' = \frac{1.1 \times 10^{-5} R i^2}{a\gamma f^{3/2}} \cos\left(2pt - \frac{3\pi}{4}\right). \qquad (15'a)$$

In considering how the efficiency of the process depends on the constants of the strip, we note that it is advantageous to make the resistance R as large as possible, and the heat capacity $a\gamma$ as small as possible. The advantage of thinness is plain.

In calculating the intensity of a sound wave, or the rate of flow of energy in the medium it is necessary to know the square of the particle velocity; and this is, from (15a) (using superimposed direct current)

$$\dot{\xi}^2_{\max} = p\xi_{\max} = \frac{4.0 \times 10^{-9} R^2 I_0^2 i^4}{a^2\gamma^2 f^3}. \qquad (16)$$

Similarly from (15′a), for alternating current,

$$\dot{\xi}'^2_{\max} = \frac{1.2 \times 10^{-10} R^2 i^4}{a^2\gamma^2 f^3}. \qquad (16')$$

These equations enable us to find the strength of the source; and knowing this, we can calculate the intensity of the sound at any distance from the source, in the ideal case in which energy is propagated in the form of spherical waves in a homogeneous medium.[1]

Since the dimensions of the source are small compared with the wave-length of sound, we may consider the strip as equivalent to a small sphere of the same area ($2a$) and which produces the same fluid velocity $\dot{\xi}$ at the surface. The velocity potential for the resulting spherical distribution of sound waves is

$$\varphi = -\frac{2a\dot{\xi}_{max}}{4\pi r}\cos\left(pt - \frac{2\pi r}{\lambda}\right), \tag{17}$$

in which $2a\dot{\xi}_{max}$ is the strength of the source, or maximum rate of emission of fluid at the source. In order to calculate the intensity of the sound produced, we make use of the two following equations

$$\text{Intensity} = \frac{W}{t} = \frac{\Pi^2_{max}}{2\rho_0 c}, \tag{18}$$

$$\Pi = -\rho_0 \frac{d\varphi}{dt}, \tag{19}$$

in which Π is the pressure change at any point in the field, c the velocity of sound, and ρ_0 the mean density of the medium. Substituting (17) in (19) we obtain for Π in terms of $\dot{\xi}_{max}$

$$\Pi = \frac{-\rho_0 a\dot{\xi}_{max}\cdot 2\pi f}{2\pi r}\sin\left(pt - \frac{2\pi r}{\lambda}\right),$$

and for the intensity, according to equation (18)

$$\frac{W}{t} = \frac{\rho_0 a^2\dot{\xi}^2_{max}\cdot f^2}{2cr^2}, \tag{20}$$

or finally in terms of the electrical energy used in the strip (direct current case)

$$\frac{W}{t} = \frac{2 \times 10^{-9} R^2 I_0^2 i^2 \rho_0 f}{cr^2\gamma^2}, \tag{21}$$

or, for alternating current,

$$\frac{W}{t} = \frac{6.0 \times 10^{-11} R^2 i^4 \rho_0 f}{cr^2\gamma^2}. \tag{21'}$$

[1] The solution here given for intensity in the case of ideal spherical distribution may easily be applied to the more practical case in which the small thermophone element is placed close to an infinite rigid plane wall. In this case, the velocity potential on the thermophone side of the wall will be twice as great as given by (17) and the intensity four times as great as given in (20).

Thus the actual intensity at any point some distance away from a thermophone whose power input is constant should increase with the first power of the frequency, and decrease with the square of the distance r. It is independent of a, the area of the strip.

Second Case: Production of Pressure Changes in Small Enclosure.—Let us assume that the strip is placed in an enclosure the dimensions of which are small compared with the acoustic wave-length, and further that the shortest distance from the strip to the boundary is large compared to the wave-length of the heat wave originating at the surface of the strip. These conditions are readily satisfied for all ordinary acoustic frequencies. If the temperature variation of the strip is given by

$$T' \sin \omega t$$

the temperature variation at any near-by point in the enclosure is

$$T_x' = T'\epsilon^{-\alpha x} \sin (\omega t \pm \alpha x). \tag{8}$$

We can consider that both sides of the strip, each of area a, give rise jointly to the temperature wave; also that this temperature wave travels a mean distance x before striking boundary defined by the equation

$$\bar{x} = \frac{V_0}{2a},$$

where V_0 is the volume of the enclosure. The alternating temperature averaged over the whole enclosure is then

$$\delta T = \frac{2a}{V_0}\int_0^{\bar{x}} T_x' dx = \frac{2aT'}{V_0}\int^{\bar{x}} E^{-\alpha x} \sin (\omega t - \alpha x) dx. \tag{22}$$

The thermal conductivity of the gaseous medium varies as the square root of the absolute temperature, while the specific heat per unit volume is practically constant at constant volume, so that the diffusivity is

$$K = K_0\sqrt{\frac{\theta_1}{273}}.$$

In terms of K_0, the diffusivity at 0° Centigrade, θ_1 is the absolute temperature of the gas near the element, this being approximately the same as the temperature of the element itself.

We then have

$$\alpha = \sqrt{\frac{\omega}{2K_0}\sqrt{\frac{273}{\theta_1}}}. \tag{23}$$

As α varies only as the fourth root of $\theta_1/273$, and conditions are easily arranged so that the temperature of the gas is not excessive, α may be considered constant in the evaluation of the integral in (17). Integrating,

$$\delta T = \frac{aT'}{V_0\alpha}\left[\epsilon^{-(V_0\alpha/2a)}\left\{\sin\left(\omega t + \frac{V_0\alpha}{2a}\right) - \cos\left(\omega t + \frac{V_0\alpha}{2a}\right)\right\} - \sin\omega t - \cos\omega t\right].$$

Now K_0 is of the order of unity ($K_0 = 1.5$ for hydrogen and .23 for air, using specific heat at constant volume) so that α (equation 23) is large for all acoustic frequencies. We may, therefore, neglect $\epsilon^{-(V_0\alpha/2a)}$ and write

$$\delta T = -\frac{\sqrt{2}\,aT'}{V_0\alpha}\sin\left(\omega t - \frac{\pi}{4}\right)$$

and, substituting the value of α from (23)

$$\delta T = -\frac{2aT'}{V_0\sqrt{\omega}}\sqrt{K\sqrt{\frac{\theta_1}{273}}}\sin\left(\omega t - \frac{\pi}{4}\right). \tag{24}$$

If the walls of the boundary are rigid, we have for a perfect contained gas, $\delta V = 0$ and the pressure change in terms of temperature change is

$$\Pi = \frac{P}{\theta_2}dT$$

if P = total pressure and θ_2 is the mean temperature of the gas. Substituting δT from (24) we have for pressure change in the enclosure

$$\Pi = \frac{2aT'P\sqrt{K_0\sqrt{\theta_1/273}}}{\theta_2 V_0\sqrt{\omega}}\sin\left(\omega t - \frac{\pi}{4}\right), \tag{25}$$

in terms of temperature variation in the strip. When direct current is used with the A.C. this is given by (7); substituting this expression for T' and dropping the dissipation factor β, we have, ($\omega = p$)

$$\Pi = \frac{.086RI_0iP\sqrt{K_0\sqrt{\theta_1/273}}}{\gamma V_0\theta_2 f^{3/2}}\sin\left(pt - \frac{3\pi}{4}\right) \tag{26}$$

and when the strip is actuated only by alternating current, we have from (25) and (7′), dropping β as before, and noting that $\omega = 2p$,

$$\Pi = \frac{.0106Ri^2P\sqrt{K_0\sqrt{\theta_1/273}}}{\gamma V_0\theta_2 f^{3/2}}\cos\left(2pt - \frac{3\pi}{4}\right). \tag{26′}$$

In (26′) f is the frequency of the alternating current and half the acoustic frequency.

Equations (26) and (26′) are in the most convenient form for calculating the stress exerted on any part of the boundary, which may be the exposed face of a sound detecting mechanism, as for example the ear. The intensity of the sound produced in the enclosure can easily be computed from the usual equations

$$\frac{W}{t} = \tfrac{1}{2}\rho_0 c^3 s^2 = \tfrac{1}{2}\frac{\Pi^2}{\rho_0 c}, \tag{27}$$

in which s = maximum condensation (Π/P), Π = maximum pressure change, ρ_0 = mean density, and c = velocity of sound in medium. Substituting the value of Π from (26) in (27), the intensity is, in the case of direct current operation

$$\frac{W}{t} = \frac{3.7 \times 10^{-3} R^2 I_0^2 i^2 P^2 K_0 \sqrt{\theta_1/273}}{\rho_0 c \gamma^2 V_0^2 \theta_2 f^3}, \tag{28}$$

and in the case of alternating current only, from (26′)

$$\frac{W}{t} = \frac{1.1 \times 10^{-4} R^2 i^4 P^2 K_0 \sqrt{\theta_1/273}}{\rho_0 c \gamma^2 V_0^2 \theta_2^2 f^3}. \tag{28′}$$

It is seen from these equations that the intensity in this case is inversely proportional to the cube of the frequency. The temperature θ has been retained in equations (28) and (28′), and the calculation has been carried through to a determination of the intensity; but there is not much difference between equations (21), (21′) which deal with the intensity in the first case, and (28), and (28′) which deal with the intensity in the second case, except the frequency-variation law.

In all cases the temperatures of gas and of strip must be taken into account; and in most cases it is possible to arrange experimental work and calculation so that this can be done in a very simple way.

Experimental Tests.

The first test that was made was a rough verification of equations (15*a*) and (26) to see if the computed effect was of the right order of magnitude. The method used consisted in setting the thermophone and an electro-mechanical source (ordinary telephone receiver) for equal intensity at the same pitch, and measuring the electrical input into each instrument. The setting for equal intensity was made with the unaided ear, for simple experiments have shown that the ear judges equality between two tones of the same pitch to within 4 or 5 per cent.[1] The telephone receiver had previously been calibrated as a sound generator by measuring the motion of the diaphragm with a microscope when a known value of alternating current was sent through it. In the case of the vibrating telephone diaphragm, the motion of the diaphragm is greatest near the center, falling off to zero at the edge. The law of

[1] Or to one per cent. under favorable conditions. The ear seems to be about as good in these measurements as the eye is in the analogous photometrical case.

distribution of amplitude over the diaphragm is, for small vibrations (at the particular frequency used), such that the bowed diaphragm may be considered from the standpoint of air displacement as replaced by a piston whose area is 0.306 that of the diaphragm, and which moves back and forth with an amplitude equal to the amplitude of the diaphragm at the center.

The data of this experiment were:

Frequency, 800.

Constants of telephone receiver:

Area of diaphragm, 18.3 sq. cm.

Effective area, 5.5 sq. cm.

800-cycle current, 1.7×10^{-5} amp.

Amplitude at center of diaphragm 1.85×10^{-6} cm.

Constants of thermophone element:

Material, platinum, of thickness 7×10^{-5} cm.

Area $a = 0.8$ sq. cm.

Effective area $2a = 1.6$ sq. cm.

$\gamma =$ (thickness times specific heat per unit volume) $= 5 \times 10^{-5}$

Resistance 1.0 ohm

Direct current $I_0 = 1.2$ amperes.

800-cycle current $= 5.6 \times 10^{-2}$ amp.

The amplitude ($\xi_{\max}$) is computed from (15*a*) corrected for temperature as per (14):

$$\xi_{\max} = \frac{6.4 \times 10^{-5} R I_0 i \cdot \sqrt[4]{273}}{a \gamma f^{3/2} \sqrt{\theta}}. \qquad (15b)$$

Allowing for a temperature of about 150° centigrade ($\theta = 423$), we compute

$$\xi_{\max} = 4.2 \times 10^{-6} \text{ cm.}$$

In comparing the acoustic outputs from these two sources, we shall assume that they are two pistons which communicate their amplitudes of motion to the adjacent medium. The strength of each source should be proportional (at fixed frequency) to the area of the piston times the amplitude of its motion. In the case of the telephone receiver, this quantity is $5.5 \times 1.85 \times 10^{-6} = 1.02 \times 10^{-5}$ cm.^{3}; and in the case of the thermophone element, $1.6 \times 4.2 \times 10^{-6} = 0.67 \times 10^{-5}$ cm.^{3} In these experiments the thermophone element was fitted into a receiver case, similar to that of the telephone receiver, and both instruments were held *loosely* to the ear. Assuming them to be *tightly* held it would be more correct to compute, instead of displacement, the relative pressure changes in the enclosed volume of air, (V_0) in order to compare the two sources. In the case of the telephone receiver the pressure change would be

$$\frac{\Pi}{P} = \frac{1.02 \times 10^{-5}}{V_0}$$

and for the thermophone, using equation (26)

$$\frac{\Pi}{P} = \frac{0.89 \times 10^{-5}}{V_0}.$$

The agreement between the two values, computed in either way is fairly good, considering the number of factors that have to be taken into account in making the comparison.

A second experimental test was made for the purpose of verifying the intensity-frequency relation given in equation (28). Ear comparison of intensities was again resorted to, the energy from the strip conductor being compared with that from a special telephone receiver at various frequencies. (The dynamical characteristics of the telephone receiver had been roughly determined so that it was possible to regulate it for equal acoustic output at various frequencies by adjusting the alternating current input.) The A.C. power input i^2R in the strip was measured for equal intensity at several frequencies, and the results are shown in Fig. 3.

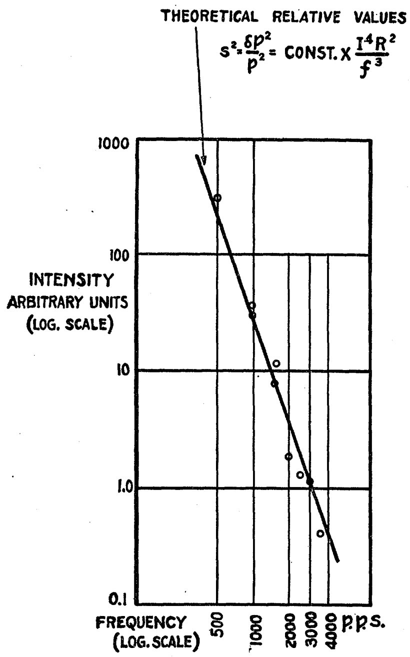

Fig. 3.

Intensity-Frequency Relation in Enclosure.

The points represent the relative intensity at different frequencies for equal A.C. power input, and are proportional to the reciprocal of the power input for equal intensity at each frequency. The curve represents the theoretical decrease in intensity according to the cube of the frequency, and the general result is a confirmation of this relation.

The writers are indebted to Mr. E. C. Wente of this laboratory for an experimental method and data which afford a much more accurate and satisfactory test of the theory than the two experiments given above.[1] The thermophone element was placed in an enclosure whose

[1] This experiment was carried out by Mr. Wente in connection with work on the theory and calibration of a new phonometer which is reported on in the paper immediately following.

volume V_0 was about 45 cubic centimeters; one of the walls of which consisted in a phonometer or pressure-measuring instrument as shown in Fig. 4*a*. This wall yielded so little that the experiment can be considered as carried out rigorously under constant volume. The pressure change in this case, if only alternating current is used to actuate the strip, is given by equation (26′). The experiments were made at a frequency of 20 cycles, the (platinum) strip being made sufficiently heavy to give a large value of thermal inertia γp so that the dissipation term β could be neglected. In order to eliminate an absolute calibration of the phonometer, a second experiment was made, using the piston apparatus shown in Fig. 4*b*, at the same frequency. The maximum pressure change Π as produced by the piston is easily calculated from mechanical considerations, and the comparison is easily made.

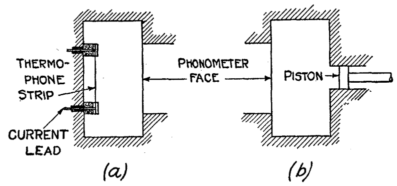

Fig. 4*a*. Fig. 4*b*.

When the piston apparatus was used, the ratio of phonometer reading to calculated pressure increase was 2.02 arbitrary units; and when the strip conductor was used, the ratio of the phonometer reading to pressure change as calculated from (26′) was 1.92 on the same scale. This confirmation to within 5 per cent., was the best we have had of the theory given in this paper.

The results obtained with platinum show that good quantitative work can be done with the thermophone when this material is used for the element. However, it is possible to obtain other materials, such as gold leaf, which are much thinner than bolometer platinum—and which are therefore very useful in cases where higher efficiency is needed. Caution should be used in applying the theoretical formulæ to elements of gold leaf since the heat capacity of gold leaf seems to be very different in different samples. Any such variations, due perhaps to absorbed gases, may be cared for (as shown by E. C. Wente in the following paper) if a check can be made against a platinum element in the same atmosphere. The correction factor thus obtained should hold for all frequencies so long as the gold foil is not unduly heated.

Comparative Value of the Thermophone as a Laboratory Source of Sound.

With regard to efficiency the thermophone compares favorably with electromagnetic and electrostatic devices except in the vicinity of their natural frequencies. In certain work it is essential that the response should be as nearly uniform as possible over a wide range of frequencies and that the relative response should be easily determinable. For such work the advantages of the thermophone are evident, for while its response diminishes with increasing frequency the law of variation is simple. When sound of indeterminate loudness and of one frequency only is desired the volume obtainable from the thermophone does not compare favorably with that from resonant mechanical devices.

The thermophone is particularly adapted to laboratory purposes because it requires no adjustment. It is extremely simple in structure and the units are readily reproducible. The determination of the acoustic effect of the thermophone depends principally upon the thermal properties of materials and is remarkably simple as compared with corresponding determinations for resonant apparatus, which usually involve motions of complicated mechanical systems. In addition, the response of the thermophone is uniform through indefinite periods of time and is not subject to the trouble of accidental detuning, which so often occurs in resonant apparatus.

Possibly even more important than the ease of determination of the sound effects in the air close to the element is the fact that these sound effects cannot react appreciably upon the source of energy whence they arise. Whenever a vibratory system is used it is always subject to reactions which may present serious complications. The thermophone seems the nearest equivalent to an ideal piston source at present obtainable.

Various modifications of size, shape and electrical resistance of the thin conductor employed may be necessary in experimental work. These need change the theory given in no essential way. On account of its simplicity from theoretical and practical points of view we believe that the thermophone in conjunction with a suitable supply of alternating current will be of material value as a precision source of sound.

Summary.

1. A description of a simple thermophone structure is given together with the theory of its operation.

2. An account is given of experimental tests the results of which are substantially in accord with the theory.

3. The thermophone is adapted to two classes of service (*a*) as a precision source of sound at any frequency (*b*) as a source of sound of known relative loudness at different frequencies throughout the acoustic range.

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