On the Frequency of Vibration of Circular Diaphragms

1922 Proc. Phys. Soc. London 35 170

(http://iopscience.iop.org/1478-7814/35/1/321)

XVII. *On the Frequency of Vibration of Circular Diaphragms.*

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RECEIVED JANUARY 24, 1923.

(COMMUNICATED BY SIR ERNEST RUTHERFORD F.R.S.)

ABSTRACT.

The Paper describes measurements of the natural frequency of diaphragms of various sizes having a rigid rim and a central boss for attachment of a microphone or receiver. In air the frequency was found by means of a monochord, but under water the resonance-frequency was noted in the neighbourhood of a subaqueous transmitter of variable pitch. The results are in good agreement with the theoretical conclusions of Prof. H. Lamb.* In many cases harmonics were observed round about a fifth or an octave above the fundamental, but their occurrence was capricious, and their pitch inconsistent with theory. The resonance peaks of the frequency curves are more or less of the same area, being high and narrow or low and broad.

The effect of increasing the pressure on one side of the diaphragm was studied, and the pressure-displacement curve was found to be linear up to the elastic limit, while the pressure-frequency curve is of the saturation type. A large diaphragm is less affected by pressure (and therefore by immersion in deep water) than a small one of the same natural frequency, in consequence of its greater thickness.

PART I.

THE object of the research described in the following Paper has been to make an experimental investigation of the theory of vibration of circular diaphragms and also to study the vibrations of such diaphragms under various conditions.

In a recent publication,* Prof. H. Lamb has given an account of a mathematical investigation of the frequency of vibration of thin circular diaphragms rigidly clamped at their circumference; he has considered the vibration of these diaphragms under various conditions in air and water and has deduced certain expressions by means of which the frequency of a diaphragm of any given material can be readily calculated.

The diaphragms used in our experiments were turned out of solid ingots of the metal (steel or bronze) with a rim ½ in. broad round the circumference, and with a small boss at the centre for the attachment of a detector.

It will be seen therefore that the conditions are only approximately those assumed in Prof. Lamb's calculations, but it was shown that variations in the rigidity of the clamping of diaphragms with a rim of the dimensions used did not produce a variation of more than ¼ per cent. in the frequency. The boss at the centre of the diaphragm also produced a variation in the frequency for which a correction had to be applied before the results could be compared. For ease in calculation of this correction the boss was kept of the same size in all the diaphragms considered, with the exception of the 6½ in. diameter diaphragms, which were of a special type. The essential parts of this type of diaphragm are shown to scale in Fig. 1, and it will be observed that it was not necessary to attach this diaphragm to any holder while making the observations, as it was itself sufficiently massive. The smaller diaphragms were, for testing purposes, clamped down rigidly to a massive steel holder, illustrated in Fig. 2.

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* Proc. Roy. Soc., A., Vol. 98, p. 205 (1920).

EXPERIMENTAL METHODS.

A very accurate measurement of the frequency of vibration of a diaphragm in air was possible by simply striking the diaphragm with a soft rubber hammer and tuning a monochord to the same pitch. In order to test the accuracy of this method, the diaphragms were excited electrically by a small electro-magnet, through which an alternating current of controllable frequency was passed.

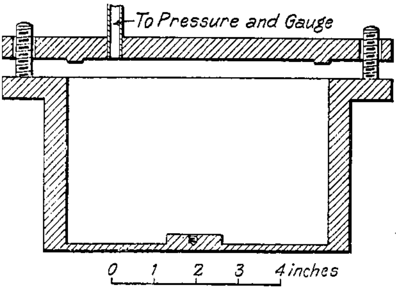

FIG. 1.—STANDARD $6\frac{1}{4}$IN. DIAPHRAGM WITH COVER PLATE. (Cover raised slightly.)

It was found that in every case these two methods gave values for the frequency which did not differ by more than 0·5 per cent. in the most extreme case.

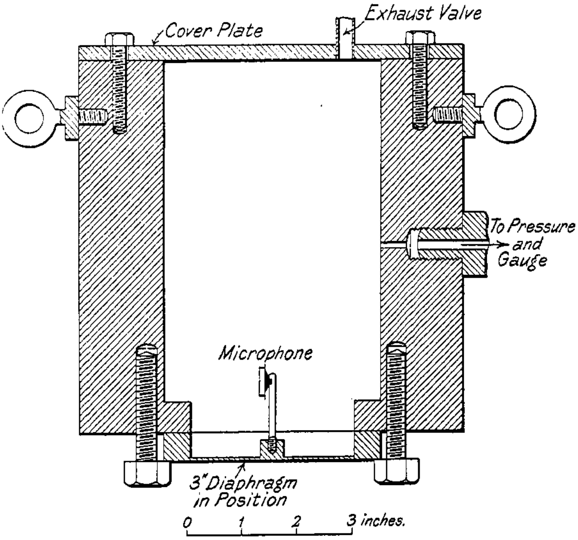

FIG. 2.—DIAPHRAGM HOLDER FITTED FOR PRESSURE EXPERIMENTS.

In water the method was not so simple, for the damping produced by the water made it much more difficult to estimate the pitch of the note emitted by the diaphragm. The method adopted was therefore to lower the diaphragm in its holder into water, in a tank constructed for the purpose, so that the diaphragm was immersed with one side in contact with the water. It was then excited by the vibrations transmitted through the water from a telephone sounder also immersed in the tank. It was found that the resonance frequency of a diaphragm in water was independent of the depth of immersion provided the depth was not less than a few millimetres, and provided the depth was not great enough for the pressure of the water to produce an appreciable distortion of the diaphragm. As the total depth of the water in the tank was about two feet, this difficulty did not arise, and for the purposes of these experiments measurements were always taken at a depth of from 6 in. to 10 in.

The vibrations of the diaphragm were examined by the use of a Brown microphone or telephone attached to the boss at the centre of the diaphragm, as shown in Fig. 2. The microphone acts as a small load on the diaphragm, but the telephone, the spring reed only being attached to the diaphragm, was found not to act as a load on the latter.

A simple Dolezelak* alternator was used to excite the telephone sounder in the tank, and by measuring the speed of rotation an accuracy of 0·1 per cent. was obtainable in the frequency measurements.

In all cases where microphones or telephone receivers or sounders were used, resonance points always occurred due to their natural frequencies, which were superposed on those of the diaphragm. These, however, were eliminated by comparing the resonance points using different sounders and detectors with the same diaphragm, when the diaphragm resonance points were easily identified.

In Air.—Prof. Lamb has shown that for a rigidly clamped uniform circular diaphragm of any given material vibrating in air the frequency (n) is given by the expression

$$n=\frac{p}{2\pi}=0{\cdot}4745\,\frac{hc_1}{a^2}$$

where h=thickness of the diaphragm, a=radius of the diaphragm, and where c is the velocity of propagation of extensional waves in an infinite thin plate of the same material and thickness and is equal to

$$\sqrt{\frac{E}{(1-\sigma^2)\,\rho_1}}$$

For an iron plate $E=2\times10^{12}$, $\rho_1=7{\cdot}8$, $\sigma=0{\cdot}28$ c.g.s. units, and we have $c_1=5{\cdot}27\times10^5$ cm. per sec.

Thence
$$\frac{p}{2\pi}=2{\cdot}50\,\frac{h}{a^2}\times10^5 \qquad \text{. . . . . (1)}$$

In Water.—The effect of water is virtually to increase the inertia in the ratio $(1+\beta)$, where $\beta=0{\cdot}6689\,\frac{\rho}{\rho_1}\,.\,\frac{a}{h}$, ρ being the density of water.

The frequency of a diaphragm with water on one side only, becomes therefore

$$\frac{p}{2\pi}=\frac{0{\cdot}4745}{\sqrt{1+\beta}}\,\frac{hc_1}{a^2} \qquad \text{. . . . . (2)}$$

With water on both sides, the value of β is doubled.

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* F. Dolezelak Zeits. Instrumentenk., XXIII., 240 (1903).

CORRECTION FOR LOAD.

If a small load on is added to the centre of a diaphragm its effect is to increase the kinetic energy of the plate. In the Paper previously referred to, Lamb gives the kinetic energy of an unloaded plate as being equal to that of a mass $M/5$, concentrated at the centre, where M is the total mass of the diaphragm. Consequently a small additional mass m at the centre is equivalent to a uniformly distributed load of $5m$. The kinetic energy is therefore increased in the ratio $\left(1+\frac{5m}{M}\right)$.

The frequency of a loaded diaphragm in air is therefore

$$\frac{p}{2\pi}=\frac{0{\cdot}4745}{\sqrt{1+\frac{5m}{M}}}\cdot\frac{hc_1}{a^2} \quad \ldots\ldots\ldots\ldots \quad (3)$$

and with water on one side

$$\frac{p}{2\pi}=\frac{0{\cdot}4745}{\sqrt{1+\beta+\frac{5m}{M}}}\cdot\frac{hc_1}{a^2}, \text{ where } m \text{ is small} \quad . \; . \quad (4)$$

Expressing $(1+\beta)$ as a load on the diaphragm it follows that the quantity of water moving with the diaphragm is equal in volume to a hemisphere whose diametrical plane is the surface of the diaphragm.

The experimental conditions therefore may be assumed not to differ materially from the theoretical in which an infinite medium is postulated.

EXPERIMENTAL RESULTS.

1. *Diaphragms in Air.*—The results obtained by the method already described may be summarised in the following table :—

TABLE I.—*Steel Diaphragms.*

Diameter of plate in inches.	Thickness of plate in mm.	Observed values.	Calculated values.		
		Frequency loaded in air. (Measured.)	Frequency unloaded in air. (Calculated.)	Frequency loaded.	Load in grams
3	1·0	1,416 ∾	1,722 ∾	1,420 ∾	4·3
3	1·5*	2,520	2,580	2,580	Zero.
4	1·5	1,350	1,450	1,330	4·3
4	2·0	1,810	1,940	1,790	4·3
6½	3·0	953	1,100	942	36·3
6½	4·0	1,330	1,475	1,310	36·3
6½	4·5	1,500	1,650	1,500	40·0
6½	6·0	1,922	2,195	1,895	67·0
7	5·0	1,464	1,585	1,450	36·3
7	6·0	1,730	1,900	1,766	36·3
7	6·6	1,902	2,095	1,958	36·3

* This diaphragm was soldered to its rim and was not turned out of a solid ingot.

A large number of other diaphragms were also examined, including many of intermediate size—but the results were in general within the limits of accuracy indicated in the typical results quoted in the above table.

Irregularities were sometimes found in individual diaphragms, but these were undoubtedly due to alteration in the elastic constants of the material due to stresses set up during the turning process.

(2) *Diaphragms with One Side in Water and the Other Side in Air.*

A similar series of measurements was made with the same diaphragms supported in their holders below the surface of the water in the tank. The following table gives the results obtained for typical diaphragms :—

TABLE II.

Diameter of Plate in inches.	" a " Radius in centimetres.	Thickness of Plate in millimetres.	Frequency with water on one side.			Load, in grams.
			Observed.	Calculated.		
			Loaded.	Unloaded.	Loaded.	
3	3·815	1·0	788	825	770	4·3
3		1·5	1,348	1,448	1,358	4·3
4	5·08	1·5	...	675	658	4·3
4		2·0	956	1,018	993	4·3
6½	8·25	3·0	535	602	543	67·4
6½		4·0	910	884	840	36·3
6½		4·5	1,000	1,028	983	36·3
6½		6·0	1,545	1,488	1,425	36·3
7	8·9	5·0	1,066	995	961	36·3
7		6·0	1,250	1,263	1,222	36·3
7		6·6	1,450	1,430	1,383	36·3

It will be observed from the preceding tables that a close agreement exists between the values of the frequencies of diaphragms calculated by Prof. Lamb and those actually found by direct measurement, and for diaphragms vibrating in air the expressions given can be taken as giving the true frequency within 2 per cent. In the case of diaphragms vibrating with one side immersed, a similar agreement between theory and experiment was observed particularly with the smaller type of diaphragm.

With the thicker 6½ in. and 7 in. types, however, it was found that the observed frequencies were all considerably higher than was anticipated from the theory. A closer examination of the frequency of a large number of 6½ in. diaphragms was therefore carried out with the results plotted in Fig. 3. Curves *A* and *B* represent the theoretical variation with thickness of the frequencies in air unloaded and with 56 grs. load. *C* shows the variation in water with the same load.

The values of the frequencies actually found in water are plotted and lie on a line corresponding more nearly to a value of

$$\beta = 0{\cdot}546\ \frac{\rho}{\rho_1} \cdot \frac{a}{h} \text{ rather than } \beta = 0{\cdot}669,\ \frac{\rho}{\rho_1} \cdot \frac{a}{h}$$

The thickness of each diaphragm was measured with a large micrometer to the nearest $\frac{1}{10}$ millimetre, but by calculating indirectly from the frequency in air it was possible to estimate the thickness to 0·01 mm. This method, however, was only

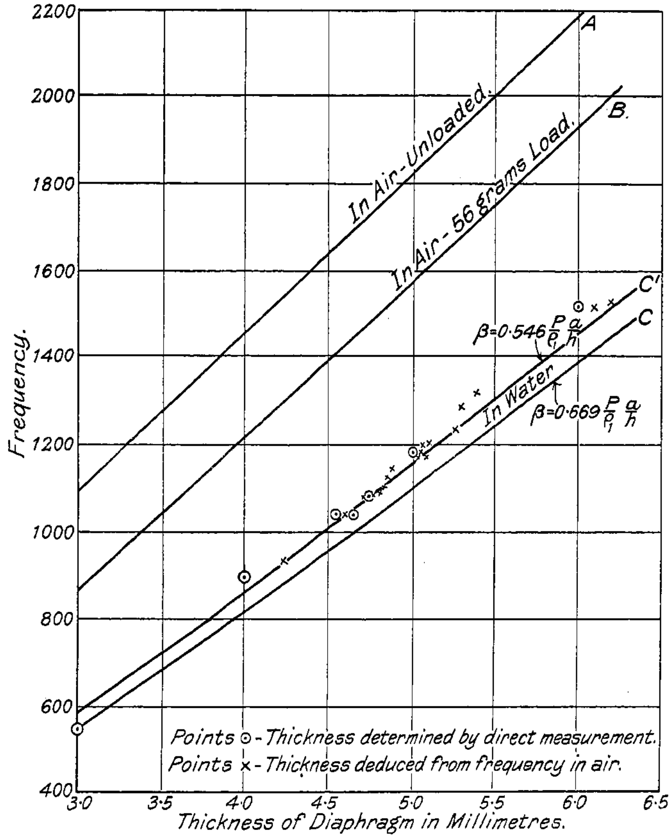

FIG. 3.—FREQUENCY OF 6½IN. DIAPHRAGMS IN AIR AND WATER.

used when it was required to produce a diaphragm accurately tuned to a particular frequedcy.

EFFECT OF LOAD.

The effect of adding a load at the centre of a diaphragm was fully examined and in this case the value for the increase in the inertia was found to agree remarkably closely with that found experimentally. Fig. 4 illustrates the effect of applying additional loads up to 113 grams at the centre of a 6½ in. diaphragm whose normal load is 67 grams. The loads consisted of flat discs of lead of diameter equal to that of the boss and screwed tightly down upon it. The values of the frequency anticipated from theory are plotted as a continuous curve while the experimental values are given as points—and the close agreement between theory and experiment in this case is at once apparent.

HIGHER HARMONICS.

Attempts were made to observe the more complicated modes of vibration of the diaphragms, but owing to very considerable variations in the behaviour of individual diaphragms it was impossible to formulate a definite rule. However, as a result of a great number of observations of many diaphragms, a first harmonic frequently occurs in water at a frequency 1½ times that of the fundamental, and a second harmonic at about double the frequency. These frequencies do not fit in with Prof. Lamb's calculations either for vibrations with a nodal circle or nodal diameter, which give much higher values, but as the occurrence of upper harmonics was very capricious, an agreement was hardly to be expected, particularly when a relatively heavy boss was attached to the centre of each diaphragm and must have exerted considerable control on its mode of vibration.

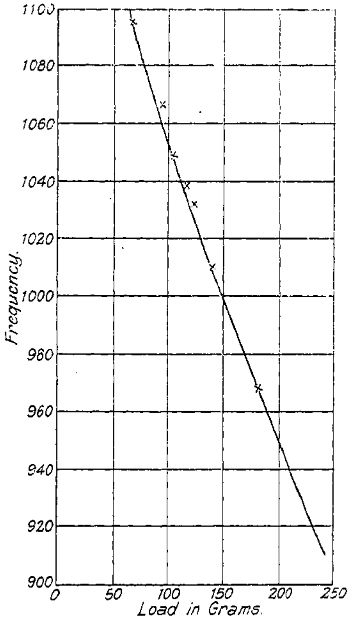

FIG. 4.—EFFECT OF LOAD AT CENTRE OF 6½IN. DIAPHRAGM.

PART II.

ON THE EFFECT OF PRESSURE ON THE VIBRATION OF DIAPHRAGMS.

The object of the present research has been to determine to what extent the frequency and general behaviour of a diaphragm as a receiver of sounds in water is governed by its depth of immersion, in other words, by the external pressure applied.

The diaphragms examined were those described in detail in the first part of this Paper, and for testing purposes were clamped into their massive steel holder—which was covered by a thick steel plate and compressed air admitted, the pressure being read by means of a gauge as indicated in the diagram Fig. 2. The large 6½ in. diaphragms of the special massive type already described were also tested and were simply arranged for the application of pressure by screwing on the stout cover plate forming part of the diaphragm unit.

The method of investigating the frequency of the diaphragms was precisely that described in Part I, the apparatus being lowered below the surface of the water in the tank and excited by the sounder of controllable frequency exactly as before.

DISTORTION OF DIAPHRAGMS UNDER STATIC PRESSURE.

The bulging of the diaphragms under static pressure was studied by means of an index attached to the central point, the motion of which was observed through a reading microscope.

The displacement of the centre of different diaphragms at various pressures is shown in the following table:—

Diaph.	Zero Pressure.	5 lb.	20 lb.	35 lb.	50 lb.
Steel—					
3 in. 1·0 mm. ...	0	0·083 mm.	0·257 mm.	0·431 mm.	0·647 mm.
3 ,, 1·2 ,, ...	0	0·058 ,,	0·117 ,,	0·175 ,,	Distorted
4 ,, 1·0 ,, ...	0	0·20 ,,	0·83 ,,	1·35 ,,	1·80 mm.
4 ,, 1·2 ,, ...	0	0·13 ,,	0·46 ,,	0·74 ,,	Distorted
4 ,, 1·5 ,, ...	0	0·07 ,,	0·25 ,,	0·45 ,,	Distorted
4 ,, 2·0 ,, ...	0	0·041 ,,	0·125 ,,	0·225 ,,	0·325 mm.
Bronze—					
3 in. 0·8 mm. ...	0	0·125 mm.	0·515 mm.	0·734 mm.	Distorted
3 ,, 1·4 ,, ...	0	0·66 ,,	0·24 ,,	0·393 ,,	0·556 mm.
4 ,, 3·0 ,, ...	0	0·033 ,,	0·108 ,,	0·183 ,,	0·257 ,,

Some of these results are shown plotted in Fig. 5, from which it will be seen that the displacement is proportional to the pressure, up to a certain point, beyond which the displacement becomes less rapid. It was found that at this pressure the diaphragm had begun to receive a "permanent set" and its resonance frequencies were permanently raised; its response to its resonance frequencies beyond this point was always found to be very much impaired. Provided a diaphragm is not strained beyond this critical point, it can be repeatedly subjected to pressure, and the displacement is always proportional to the pressure.

This is in accordance with the result given by Love,[*] who deduces the expression :—

$$\text{Displacement of centre of diaphragm } w=\frac{1}{64}\frac{Pa^4}{D}$$

where

$$D=\frac{2}{3}\frac{Eh^3}{(1-\sigma^2)}$$

h being in this case equal to half the thickness, and P the applied pressure.

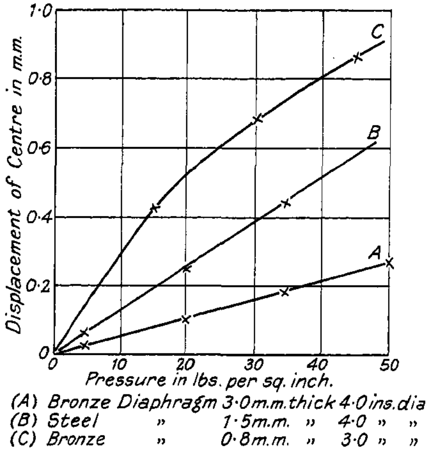

FIG. 5.

In the case of a steel diaphragm

$$w=6{\cdot}08\times10^{-9}\frac{a^4}{h^3}P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where P is expressed in lbs. per square inch.

For 4 in. 1·5 mm. steel diaphragm, w has the value 0·0238 for a pressure of 20 lb. per square inch. The value obtained experimentally in this case was 0·025 cm.

THE EFFECT OF PRESSURE ON FREQUENCY.

The curvature of the diaphragm, due to the pressure, causes the natural frequencies of the diaphragm to be raised in all cases, both in air and in water. The percentage rise in frequency due to pressure is approximately the same in air as in

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[*] Love, Mathematical Theory of Elasticity, 3rd Edition, equation (83), p. 490.

water, and the aproximate equality of the figures is shown in the following table :—

Diaphragm.	Fundamental frequency.	Percentage rise at		
		10 lb. pressure.	30 lb. pressure.	40 lb. pressure.
Steel 4 in..........	Air 1,048 ∼	1·7%	10·0%	14·3%
1·0 mm.	Water 481	2·1%	10·4%	13·6%
Steel 4 in..........	Air 1,212	1·5%	5·05%	8·8%
1·2 mm.	Water 602	0·5%	5·70%	9·9%
Steel 4 in..........	Air 1,304	To small to measure.	1·20%	1·7%
1·5 mm.	Water 892*		2·70%	3·9%
Bronze	Air 1,120	4·6%	15·4%	Distorted.
3 in. 1·0 mm. ...	Water 478	4·9%	15·7%	Distorted.

* In this case the 1st harmonic was observed.

The figures for the fundamental frequency given in the second column of the above table are the actual experimental figures. The effect of load is allowed for in calculating the percentage rise.

The percentage rise of frequency for diaphragms of different dimensions is shown in the following table ; in this table the diaphragms are all made of steel :—

Diaphragm.	Rise in Frequency.				$\frac{a^4}{h^4}$	Anticipated rise in frequency in terms of 4 in. 1 mm. diaph.
	30 lb. pressure.		40 lb. pressure.			
	Percentage.		Percentage.			
	Actual.	In terms of that of 4 in. 1 mm. diaph.	Actual.	In terms of that of 4 in. 1 mm. diaph.		
3 in. 1·0 mm. ...	3·47%	33·3	7·3%	31·8	$2\cdot10\times10^6$	31·8
3 in. 1·2 mm. ...	1·34%	12·9	2·8%	12·8	$1\cdot00\times10^6$	15·2
4 in. 1·0 mm. ...	10·4%	100·0	Distorted		$6\cdot63\times10^6$	100·0
4 in. 1·2 mm. ...	5·05	48·0	8·8%	38·4*	$3\cdot20\times10^6$	48·3*
4 in. 1·5 mm. ...	2·0%	19·2	3·0%	13·1	$1\cdot31\times10^6$	19·8
6·5 3·2 mm. ...	0·64%	6·2	1·59%	6·9	$0\cdot443\times10^6$	6·7

* The discrepancy here was due to the fact that the diaphragm was just beginning to be permanently distorted.

The theory of the vibration of a circular diaphragm when under flexure is very involved and has not been worked out, but it is important to find some simple relation to co-relate the effects of pressure in different diaphragms, as found by experiment.

The simplest relation is :—

$$\text{Percentage rise in frequency} = A\frac{a^4}{h^4}P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

where A and h are expressed in centimetres and P in lbs. per square inch, A being an empirical constant depending on the material. For steel, with these units, A has a mean value deduced from a large number of trials of $6\cdot0\times10^{-8}$.

A few further experiments were carried out using bronze diaphragms and, as would be anticipated, a rise in frequency occurred which was much greater than with steel diaphragms, being 10 to 12 times as large for corresponding diaphragms of the same dimensions.

These relations are shown by the preceding table, two columns of proportional values being included for comparison. It will be observed that they do not hold when the diaphragm is strained beyond its elastic limit which is reached when the rise in frequency reaches about 10-12 per cent.; this figure seems fairly uniform for steel diaphragms of all dimensions.

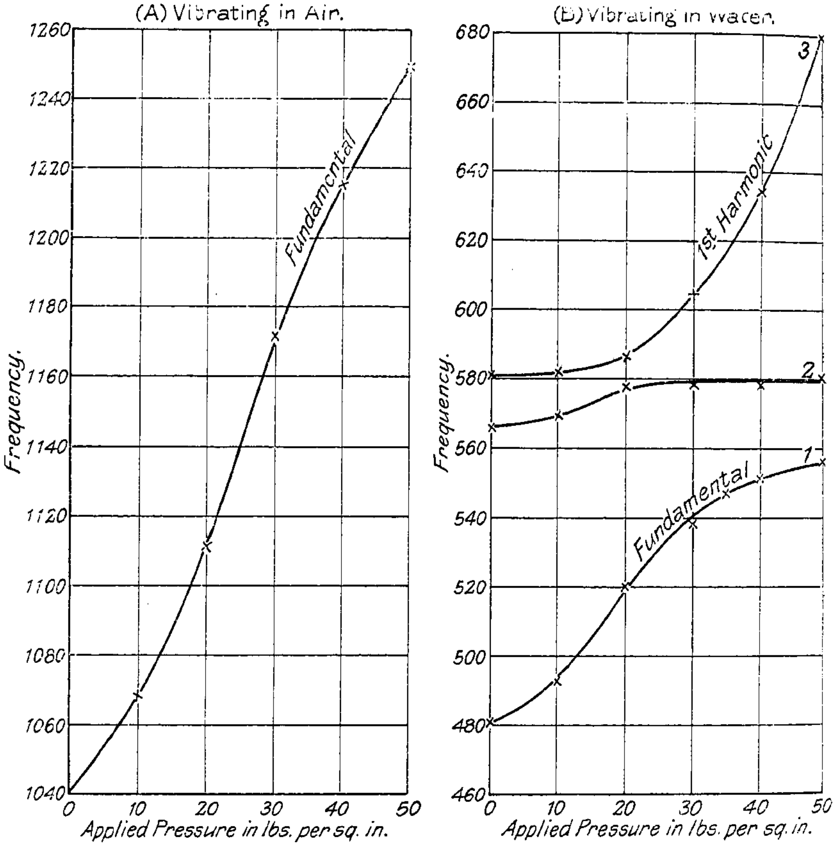

FIG. 6.—VARIATION OF RESONANCE FREQUENCY WITH PRESSURE.
Steel Diaphragm 1·0 mm. thick, 4·0 inches diameter.

GENERAL OBSERVATIONS.

In all cases the frequencies of the resonance points of a diaphragm are raised by increasing the pressure on one side of the diaphragm. (*See* typical curves, Fig. 6.)

The percentage variation in frequency with increase of pressure is the same in air as in water for the same diaphragm, and for different diaphragms, other things being equal, it is less the thicker the diaphragm.

The displacement of the centre of the diaphragm under uniform pressure is proportional to the pressure up to the point where the diaphragm begins to acquire a permanent "set" when the rate of displacement with increase of pressure begins to fall off.

When a resonance becomes diminished in loudness, it increases in area or extent, and when it increases in loudness it diminishes in extent; *i.e.*, becomes sharper, as though the sum of energy in the resonance tended to remain constant. In some cases the energy appears to distribute itself between different resonance points, one sound-maximum increasing in loudness at the same time that another is diminishing.

In addition to the fundamental and harmonics, smaller maxima were frequently present initially. These were not easily followed, as they behaved irregularly. They were no doubt due to local strains, &c., set up when the diaphragms were being turned on the lathe. They usually diminished or disappeared on the application of pressure. Curve 2, Fig. 6 (B), is an example of such a maximum. As the pressure was still further increased, this maximum became very faint and, at the same time the fundamental and first harmonic increased in loudness.

With the fundamental of all the diaphragms which have been examined, the curve showing the rise of frequency with pressure resembles a saturation curve, the position of the resonance point rising much more slowly with pressure beyond a certain critical pressure.

With first harmonics and others, the initial portions of the curve are similar to the initial portions of the curves for fundamentals, but the corresponding features occur at higher pressures. Curve 3, Fig. 6 (B), represents a typical first harmonic.

At the pressure at which the curve begins to turn to the horizontal, the diaphragm begins to acquire a permanent "set," and the amplitude of vibration diminishes.

Since the percentage rise in frequency for a given pressure and material varies as $\frac{a^4}{h^4}$ and the fundamental frequency varies as $\frac{h}{a^2}$, it follows that for diaphragms of the same material and different dimensions, of a given frequency (*i.e.*, $\frac{h}{a^2}=$constant), the percentage rise in frequency for a given pressure varies as $\frac{a^2}{h^3}$.

From this it is seen that a large (and thick) diaphragm would be less affected by pressure than a small (and thin) one of the same material and frequency. For example, a steel diaphragm of fundamental frequency 481 and dimensions 4 in. by 1 mm., rises in frequency at a pressure of 30 lb. per square inch, by 10·4 per cent., whereas a steel diaphragm of fundamental frequency 535 and dimensions 6·5 in. by 3·2 mm., rises in frequency at the same pressure by only 0·64 per cent.

These results are now published by permission of the Admiralty. The experiments were carried out for Naval purposes during the war under the close direction of Sir Ernest Rutherford, F.R.S., to whom the authors have great pleasure in acknowledging their indebtedness.

DISCUSSION.

Dr. W. S. Tucker inquired whether double resonance had been observed. He had found that it was hard to avoid this phenomenon in the neighbourhood of air cavities, which form a coupled system with the diaphragm.

Mr. J. H. Powell replied that double resonance had been observed, and that it accounted, for instance, for the form of curve 2 in Fig. 6 of the Paper. In general, however, it was avoided by suppressing air cavities.

Dr. F. Ll. Hopwood said that much work has been done on multiple resonance, the results being analogous to those found for coupled electrical circuits. The resonance points can be varied at will by altering the coupling. No satisfactory theory has been given for overtones produced under water as it has not been found possible to deal analytically with the flow of water from one segment to another vibrating in opposite phase. It is doubtful whether the breadth of the resonance peaks has much significance, since the measurements of amplitude are scarcely reliable although the frequencies are accurately known.

Dr. A. O. Rankine questioned the propriety of calling the overtones "harmonics." In modern music such relations may be of small importance, but for scientific purposes a distinction should be drawn between a perfect octave, for instance, and a mere overtone such as an approximate fifth or octave.

Prof. C. L. Fortescue suggested that the multiple resonance was due neither to harmonics nor to non-harmonic overtones, but to the coupling of two elastic systems, not necessarily including the air cavities mentioned by Dr. Tucker. The structure supporting the diaphragm would form one such system, and its being mechanically coupled to the diaphragm would perhaps account for what the authors had described as the fundamental and first harmonic in Fig. 6.

Dr. Chree, in a written communication, pointed out that the formula $p/2\pi = 0{\cdot}4745hc_1/a^2$, ascribed to Prof. Lamb, is only an approximation—though a good one—the exact value of the numerical factor being 0·4693, as stated by Prof. Lamb himself. He would have expected the more exact formula to be used. He also pointed out that "iron" being so variable a substance, it would be desirable for high accuracy to determine c_1^2 directly for the actual material employed.

Reply to the Discussion by Dr. J. H. T. Roberts : Double resonance was in some cases observed, but special efforts were always made to avoid air-bubbles, as it was found that the presence of even a minute air-bubble seriously interfered with the sensitivity of the diaphragm for the reception of sound from water. The behaviour of the resonances was also found to be capricious when there were minute bubbles adhering to the system.

The theory of the mode of vibration of a diaphragm under the conditions necessary for the production of overtones is very complicated, and it is possible that the apparent overtones observed (or at any rate, some of them) might be due to a mechanically coupled system, as mentioned by Prof. Fortescue. But it is unlikely that this is the whole explanation, as not only was this possibility foreseen, but cases were observed in which this coupled action was definitely taking place, and in consequence steps were taken to prevent it.