

face condenser, *i. e.*, heat flow from condensing steam to circulating water, but recent tests of feed-water heaters show the same peculiarity as was mentioned in the case of double pipe brine coolers—a rise of the coefficient with water speed to values close to  $U = 1000$ , although few designers would think of exceeding  $U = 350$  for these heaters. Passage of heat from condensing liquid to boiling liquid at lower temperature is a characteristic of evaporators of single and multiple effect, and for these  $U = 300$  is a fair average value. It is interesting to note that this same value also applies to the generators of absorption refrigerating machines where the heat of condensing steam is given up to rich aqua ammonia liquor.

There is probably no single class more interested, consciously or unconsciously, in increasing the effectiveness of heat transfer surfaces than chemical manu-

facturers, and there is likewise no class with the opportunities for securing the great mass of data necessary for the formulation of laws of design with equal ease. There is presented in this issue a paper on the subject, discussing the various physical constants involved, which, while it leaves much to be said on the subject, will serve to open a thorough investigation of it in these pages. It is hoped that all the users of heat transfer apparatus will send in letters and criticisms of this paper, possibly preparing additional papers, but more important than all else, large quantities of data on every form of heat transfer apparatus in their establishments, all of which we will undertake to print as part of a campaign of improvements that is needed at least as much, if not more, than any other single thing common to all interests.

C. E. LUCKE.

## ORIGINAL PAPERS.

### HEAT TRANSMISSION.

By HAROLD P. GURNEY.

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It is the purpose of this paper to evolve and present a rational method of analyzing problems in heat transmission along the lines laid down by such eminent and practical authorities as Hausbrand,<sup>1</sup> Mollier,<sup>2</sup> and Berlowitz.<sup>3</sup> The study of the flow or transmission of heat has important bearings in mechanical, electrical, and chemical engineering, and it is a subject which merits a more systematic treatment than is usually accorded to it.

The usual technical case of heat transmission is where heat flows from a relatively hot fluid through a separating plate to a cold fluid. The fluids are usually air, hot gases, water, or steam; in chemical work, the number is almost without limit. The plate is usually thin and of metal, and may be flat, spherical, or tubular in shape. Either or both fluids may be moved parallel or perpendicular to or against the plate, or portions of the plate.

In heat flow, the temperature gradient may be defined as the rate of change in temperature with respect to distance in the direction that the heat is moving; in ordinary units, it is the drop in Fahrenheit degrees per linear inch in the direction of heat flow. When heat is moving uniformly from fluid to fluid through a plate, the temperature gradient is constant throughout the plate. Here the temperature gradient is determined by the rate of heat flow which is constant, and the heat conductivity (or resistivity) of the plate material. In the fluid, the temperature gradient is not constant, it is a maximum at the boundary surface and drops off as the distance from the boundary or plate surface increases. In an infinitesimal film just next to the plate, the temperature gradient is determined solely by the rate of heat flow and by

the true conductivity of the fluid. Beyond this, heat flow results not only from true conductivity, but also from actual transportation of portions of the fluid in the direction of heat flow. This is called convection and it is the heat-transporting power of the fluid. The convectivity of a fluid at any point depends on: (1) the increase in density of the fluid per unit change of temperature, (2) the density, (3) the viscosity, (4) the mean distance from the fluid boundary, and (5) the velocity. It may further be observed that in the plate, the heat flow is constant, whereas in the fluid, the heat flow is greatest at the surface of the plate and diminishes to nothing at a point or points farthest from the plate.

From a practical point of view, it is usual to consider an abrupt drop in temperature from the plate surface to the fluid to exist, and to assume that the fluid has the same temperature throughout. This temperature is the mean effective temperature of the fluid, and, by definition, it is the temperature it would attain if completely mixed without gain or loss of heat. This is a temperature which is different for different parts of an apparatus, and while practically its initial and final values may be readily measured, its intermediate values are obtained with difficulty. For example, the temperature of water circulating through heating coils can be measured before it goes in and after it comes out, but the mean effective temperature at any cross section of the coils is not so easily determined.

Diagrammatically, this discussion may be made clearer by the two following distance-temperature plots. Actually the temperature follows the curve shown in A, but for convenience of treatment, it is assumed to follow the broken curve in B.

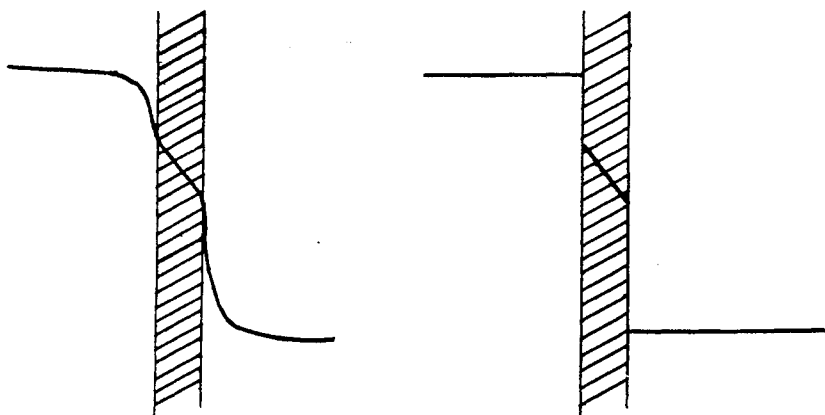
This differentiates heat conductivity into two distinct forms: (1) solid heat conductivity where heat traverses a solid or a fluid at rest; and (2) surface heat conductivity where heat passes between a solid and a fluid. The first is the internal conductivity,

<sup>1</sup> Verdampfen, Kondensieren, and Kühlen, 1909.

<sup>2</sup> Zeitschrift des Vereines deutsche Ingenieure, 1897.

<sup>3</sup> Zeitschrift für Apparatenkunde, 1908.

and the second is the boundary conductivity or external conductivity when referred to a plate separating two fluids. The first form of conductivity is a physical characteristic or property of matter. The second form depends not at all on the solid from



A

B

which heat is passing, but on the fluid and especially on such factors as (1) the conductivity, (2) the density and viscosity, (3) the expansiveness to heat, (4) the mean hydraulic radius of the space occupied by the fluid, and (5) the mean fluid viscosity.

In order to investigate quantitatively the flow of heat, a system of notation is here used which is consistently adhered to.  $Q$  is the heat units in British thermal units which flow through a heat-transmitting plate in a time  $T$  hours, where the temperature difference between the two fluids on either side is  $\Delta$  degrees Fahrenheit.  $Z$  is the heat resistance of the plate. A strict proportionality between the rate of heat flow and the drop in temperature may be assumed. The ratio of the temperature drop to the rate of heat flow may be defined as the resistance, the reciprocal of conductance.

$$z = \frac{\Delta}{Q}, \quad Q = \frac{\Delta T}{Z}$$

The resistance  $Z$  is made up of two separate component resistances: (1) the internal resistance  $Z_1$  of the plate, and (2) the external resistance  $Z_0$ , the sum and resultant of the boundary resistance of the plate.

$$Z = Z_1 + Z_0$$

The resistance possessed by a portion of the plate of one square foot area and one inch in thickness is the specific internal resistance or the internal resistivity and is designated by  $\rho$ . The area of the plate is  $A$  square feet, the thickness is  $\delta$  inches, and the internal resistance is  $Z_1$ . The plate resistance is proportional to its thickness and to the inverse of its area.

$$Z_1 = \frac{\rho \delta}{A}$$

The internal resistance of a square foot of plate of thickness  $\delta$  inches is  $z_1$ .

$$z_1 = Z_1 A = \rho \delta$$

The external resistance is the resultant and preceding sum of the boundary resistances of the two sides of the plate. The resistance of a square foot on one side is  $\zeta_1$  and on the other side is  $\zeta_{11}$ ; separate resistances of both sides are  $\zeta_1/A$  and  $\zeta_{11}/A$ , and

$$Z_0 = \frac{\zeta_1}{A} + \frac{\zeta_{11}}{A}$$

The external resistance of a square foot is  $z_0$

$$z_0 = Z_0 A = \zeta_1 + \zeta_{11}$$

and

$$Z = \frac{\zeta_1}{A} + \frac{\rho \delta}{A} + \frac{\zeta_{11}}{A}$$

The resistance of a square foot of plate both internal and external is  $z$ .

Then,

$$z = ZA = \zeta_1 + \rho \delta + \zeta_{11}$$

and

$$Q = \frac{\Delta AT}{\zeta_1 + \rho \delta + \zeta_{11}}$$

These same facts may be expressed in terms of conductances and conductivities instead of resistances

and resistivities, but it must be observed that where resistances in series are additive, conductances in series are not additive. The resultant of conductances in series is the reciprocal of the sum of the reciprocals of the separate conductances. This results from the fact that conductance is the reciprocal of resistance and is the ratio of heat flow to the temperature drop. If  $G$  be used to designate conductance, then,

$$G = \frac{Q}{\Delta} = \frac{\text{rate of heat flow}}{\text{temperature drop}}$$

hence,

$$Q = \Delta TG.$$

The conductance  $G$  is the resultant of the internal conductance  $G_1$  and the external conductance  $G_0$ .

$$G = \frac{1}{\frac{1}{G_1} + \frac{1}{G_0}}$$

The specific internal conductance or the internal conductivity is the conductance of a portion of plate of one square foot area and one inch thickness and is designated by  $\lambda$  the reciprocal of  $\rho$ . Since the internal conductance is proportional to the area of the plate and inversely as its thickness, then,

$$G_1 = \frac{A\lambda}{\delta}$$

The internal conductance per square foot is  $g_1$ .

$$g_1 = \frac{G_1}{A} = \frac{\lambda}{\delta}$$

The boundary conductances per square foot, or the boundary conductivities are  $\gamma_1 = 1/\zeta_1$  and  $\gamma_{11} = 1/\zeta_{11}$  and  $G_0$  is the external conductance.

$$G_0 = \frac{A}{\frac{1}{\gamma_1} + \frac{1}{\gamma_{11}}}$$

The external conductance of a square foot is  $g_0$ .

$$g_0 = \frac{G_0}{A} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}$$

Then the combined conductance  $G$  of the plate, both with respect to internal and external conductances is given by the following expression:

$$G = \frac{A}{\frac{1}{r_1} + \frac{\delta}{\lambda} + \frac{1}{r_2}}$$

The conductance per square foot is  $G$ ,

$$g = \frac{G}{A} = \frac{1}{\frac{1}{r_1} + \frac{\delta}{\lambda} + \frac{1}{r_2}}$$

and

$$Q = \frac{\Delta T}{\frac{1}{r_1} + \frac{\delta}{\lambda} + \frac{1}{r_2}}$$

In most heat-transmitting apparatus, the temperature drop is not the same at all points of the drop. The equations deduced hold for infinitesimal plate areas, but by adopting a mean temperature drop in the place of  $\Delta$ , they may be applied to finite areas. The maximum and minimum temperature differences may always be obtained from measurements on the temperatures of both fluids both before and after transit through the apparatus. The simplest mean temperature difference would be either the arithmetic average, or the geometric mean of the maximum and minimum, but the former is too high, while the latter is too low. The most rational mean is the logarithmic mean and it is obtained by considering the temperature difference to vary relative to time, or distance traversed at a rate proportionate to its instantaneous value. If  $\Delta_0$  is the minimum temperature difference,  $\Delta_1$  the maximum temperature difference and  $\Delta_m$  the mean temperature difference, then the latter may be expressed in terms of the two former by the following expression:

$$\Delta_m = \frac{\Delta_1 - \Delta_0}{\ln \frac{\Delta_1}{\Delta_0}}$$

Let the ratio of  $\Delta_0$  to  $\Delta_1$  be a variable,  $x$ .

$$\Delta_m = \Delta_1 \frac{1-x}{-\ln x}$$

An arithmetic mean would be  $\Delta_1 \frac{x+1}{2}$ , while a geometric mean would be  $\Delta_1 \sqrt{x}$ . The following table brings out the relations between the three means.

$x$	$\frac{x+1}{2}$	$\frac{x-1}{\ln x}$	$\sqrt{x}$
1.000	1.000	1.000	1.000
0.900	0.950	0.949	0.949
0.800	0.900	0.896	0.895
0.700	0.850	0.841	0.836
0.600	0.800	0.783	0.775
0.500	0.750	0.722	0.707
0.400	0.700	0.636	0.632
0.300	0.650	0.583	0.548
0.200	0.600	0.497	0.447
0.100	0.550	0.382	0.316
0.050	0.525	0.317	0.223
0.020	0.510	0.251	0.141
0.010	0.505	0.215	0.100
0.009	0.500	0.000	0.000

For design, the use of the geometric mean gives safer values than the logarithmic mean; for investigation, the logarithmic mean should be used. Occasionally, the arithmetic mean may be used.

Where both sides of a plate have the same area as in the case of a flat plate, no doubt arises as to the proper value of  $A$ ; but where the areas of both sides of a plate differ as in a pipe, the problem of obtaining a mean area presents itself for investigation. Since the mean heat-transmitting area is nearer in value to the area of the side where there is the greatest heat resistivity, a single mean area may be obtained by weighting the areas with the respective resistivities. The area  $A$ , has a heat resistivity,  $\zeta_1$ ;  $A_{11}$  has a heat resistivity,  $\zeta_{11}$ , and  $A_m$  is the mean area.

$$A_m = \frac{\zeta_1 A_1 + \zeta_{11} A_{11}}{\zeta_1 + \zeta_{11}}$$

For a pipe  $A_0$  and  $A_2$  being the internal and external areas, Mollier gives the following formula:

$$A_m = \frac{\zeta_1 + \zeta_{11} + \delta \rho}{\frac{\zeta_1}{A_1} + \frac{\zeta_{11}}{A_{11}} + \frac{\rho}{2} \ln \frac{A_2}{A_0}} = \frac{\zeta_1 + \zeta_{11}}{\frac{\zeta_1}{A_1} + \frac{\zeta_{11}}{A_{11}}} \text{ approximately.}$$

A rough rule is to employ as mean area the area whose resistivity is greatest, except where the areas have about equal resistivities, and then an average is quite close.

Apparatus for transmitting heat between fluids may be classified under four types: (1) counter-current, (2) parallel current, (3) perpendicular current, and (4) single current. In counter-current apparatus, the fluids move in opposite directions, and the temperature difference does not generally vary greatly. In parallel current apparatus, the fluids move in the same direction; the temperature difference is at first large and rapidly diminishing, later it is small and slowly diminishing. In perpendicular current apparatus, the fluids move in directions at right angles to each other, and in this respect stands midway between counter-current and parallel current apparatus. In practice, perpendicular current and counter-current apparatus are sometimes placed in series and so become essentially parallel current or counter-current in effect. In single current apparatus, one fluid, and usually the fluid whose temperature it is desired to modify, remains in the apparatus during the entire operation while the heating or cooling fluid moves through at a constant rate. The use of the single current apparatus makes the process intermittent; with other types, the process is continuous.

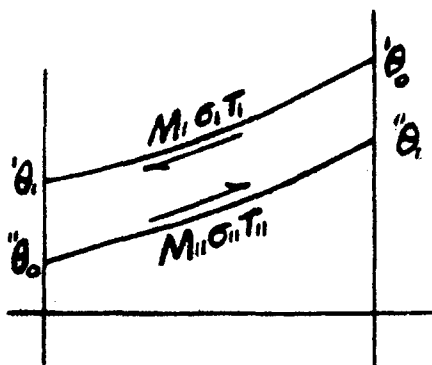
For purposes of investigation and design, all four types of apparatus may be treated in essentially the same manner.

When investigating a heating or cooling apparatus, the initial and final temperatures of both fluids are measured. From this the maximum and minimum, and consequently the mean temperature difference may be obtained. The amount of heat transmitted can be found by multiplying the weight of either fluid into its specific heat, then this into its change in temperature. If necessary, the heat lost in radiation

from the exterior of the apparatus may be taken into account. Knowing the area of the heat-transmitting plate or tubes, all data necessary to compute the heat-transmitting capacity of the walls is at hand.

When designing an apparatus, the heat-transmitting capacity of the plates must be previously estimated from data available or assumed. A preliminary assumption on this point to determine within certain limits the velocities of the fluids, and from this a closer final assumption may be made. The rate at which heat is to be transmitted is obtainable from the weight, specific heat, initial and final temperatures of the fluid whose temperature is to be raised or lowered, and the length of time assigned to the operation. With respect to the fluid supplying heat or cold, its initial temperature and specific heat may be regarded as fixed, but the amount delivered per hour and final temperature are interdependent variables. The most logical starting point in design is to make  $x$  a variable ratio of the variable temperature difference to the fixed temperature difference. For any value of  $x$  the rate of supply of heating or cooling fluid and the required surface, hence approximate size of apparatus, is easily computed. As  $x$  increases, that is, as a larger temperature difference and a smaller temperature change is obtained, the fluid supply rate becomes greater but the size of the apparatus becomes smaller. Against  $x$  as abscissae may then be plotted as ordinates the total amount of depreciation, interest, maintenance, and rental on apparatus plus the cost of fluid together with the pumping and storing of the same. The curve obtained will have a minimum, and the best value of  $x$  to adopt should be as near to this minimum as possible, at the same time taking into consideration other points of plant economy that would tend to shift this value. In the design of single current and perpendicular current apparatus, two variables similar to  $x$  may be chosen instead of one; a simple expedient in these cases is to employ, as abscissae, the products of the two variables and then later try the effect of shifting the ratio of these variables.

Owing to the variety of factors which may come into play, it is not advisable to lay hard and fast rules for the solution of problems in the design of heat-transmitting apparatus. A perfectly general mathematical method might not be at once soundly



theoretical or practical, but a brief outline of the simple mathematical aspects ought to be presented.

Two fluids move counter-current. A weight of hot fluid  $M_1$  of specific heat  $\sigma_1$  enters at  ${}^1\theta_0$  and leaves at  ${}^1\theta_1$  undergoing a cooling  $\tau_1$  degrees, while a weight of cold fluid  $M_{11}$  of specific heat  $\sigma_{11}$  enters at  ${}^{11}\theta_0$  and leaves at  ${}^{11}\theta_1$ , changing in temperature by  $\tau_{11}$  degrees. When the object is to heat the cold fluid,  ${}^1\theta_1$  is a variable and it may be made a function of  $x$  by placing  ${}^1\theta_0 - {}^1\theta_1 = \Delta$  and  ${}^1\theta_1 - {}^{11}\theta_0 = \Delta x$ . Then,

$$\Delta_m = \Delta \sqrt{x} \text{ or } \Delta \frac{x-1}{\ln x}$$

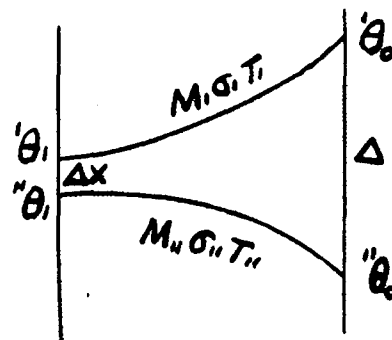
$$M_1 = \frac{Q}{\sigma_1(\Delta - \Delta x + \tau_{11})} \text{ and } A = \frac{Q}{\Delta_m T_g}$$

$Q$  is the total heat to be transferred and  $Q = M_1\sigma_1\tau_1 = M_{11}\sigma_{11}\tau_{11}$  neglecting outside radiation losses.

When the object is to cool the hot fluid,  ${}^{11}\theta_1$  is the variable and  $x = \frac{{}^1\theta_0 - {}^{11}\theta_1}{{}^1\theta_1 - {}^{11}\theta_0}$ ,  $\Delta = {}^1\theta_0 - {}^{11}\theta_0$ . Then,

$$\Delta_m = \Delta \sqrt{x} \text{ or } \Delta \frac{x-1}{\ln x}$$

$$M_{11} = \frac{Q}{\sigma_{11}(\Delta - \Delta x + \tau_1)} \text{ and } A = \frac{Q}{\Delta_m T_g}$$



Parallel current heat transfer is graphically presented above. When the object is to heat the cold fluid,  ${}^1\theta_1$  is variable; to cool the hot fluid,  ${}^{11}\theta_1$  is variable. Then,

$$\Delta_m = \Delta \frac{x-1}{\ln x}$$

$$M_1 = \frac{Q}{\sigma_1(\Delta - \Delta x - \tau_{11})} \text{ or}$$

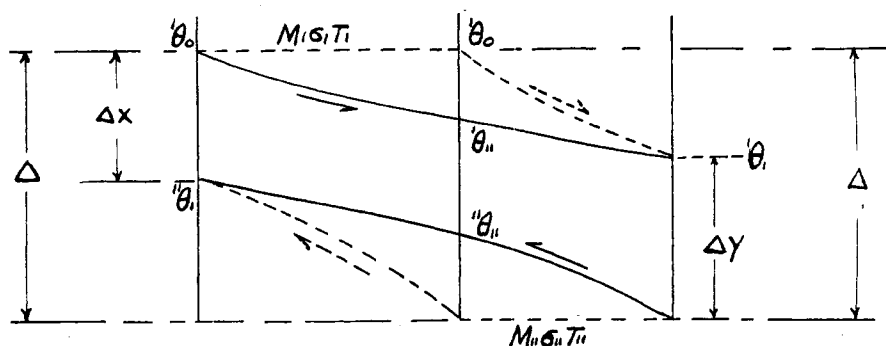
$$M_{11} = \frac{Q}{\sigma_{11}(\Delta - \Delta x - \tau_1)} \text{ and } A = \frac{Q}{\Delta_m T_g}$$

In perpendicular current heat exchange, a hot fluid of weight  $M_1$ , specific heat  $\sigma_1$ , enters at  ${}^1\theta_0$  and is cooled to a temperature ranging from  ${}^1\theta_{11}$  to  ${}^1\theta_1$  and undergoing a mean change of temperature ranging from  ${}^{11}\theta_{11}$  to  ${}^{11}\theta_1$  and undergoing a mean temperature increase of  $\tau_{11}$  degrees. As before,  $Q = M_1\sigma_1\tau_1 = M_{11}\sigma_{11}\tau_{11}$ .

$$\text{Let } x = \frac{{}^1\theta_0 - {}^{11}\theta_1}{{}^1\theta_0 - {}^{11}\theta_0} \text{ and } y = \frac{{}^1\theta_1 - {}^{11}\theta_0}{{}^1\theta_0 - {}^{11}\theta_0}$$

Then,

$$\Delta_m = \Delta \sqrt{xy} \text{ or } \Delta \frac{x-1}{\ln x} \frac{\gamma-1}{\ln \gamma}$$

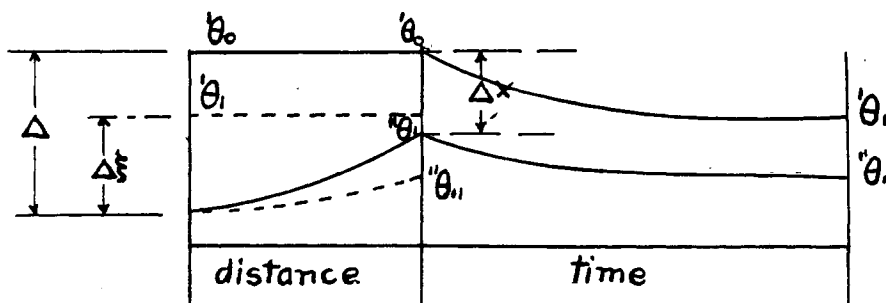


Assuming  $\theta_{11} \approx \theta_1$  and  $\theta_{11} \approx \theta_{11}$  straight lines, which is approximately true, then,

$$M_1 = \frac{2Q}{\sigma_1 [\Delta(3-x-y-\sqrt{xy}) - 2\tau_{11}]} \text{ or } M_{11} = \frac{2Q}{\sigma_{11} [\Delta(3-x-y-\sqrt{xy}) - 2\tau_{11}]}$$

and

$$A = \frac{Q}{\Delta_m T_g}$$



In a single current apparatus for cooling, a hot fluid is cooled from  $\theta_0$  to  $\theta_1$ . It is here assumed that the temperature of all parts of the fluid that remains in the apparatus during the operation is the same at any time, owing to the use of good stirring devices. The cold fluid enters at  $\theta_0$  and is heated at first to  $\theta_{11}$ , finally to  $\theta_{11}$  undergoing a mean temperature rise of  $\tau_{11}$  degrees. Again,  $Q = M_1 \sigma_1 \tau_1 = M_{11} \sigma_{11} \tau_{11}$ . Assuming the curve  $\theta_{11} \approx \theta_{11}$  a downward parabola, then,

$$\Delta_m = \Delta \sqrt{x\xi} \text{ or } \Delta \frac{x-1}{\ln x} \frac{\xi-1}{\ln \xi}$$

$$M_{11} = \frac{3Q}{\sigma_{11} \Delta (1+2\xi)(1-x)} \text{ and } A = \frac{Q}{\Delta_m T_g}$$

For an extended mathematical study of problems of this type, the reader is referred to papers by M. Berlowitz, "Beitrag zur Berechnung der Heizflächen," *Zeitschrift für Apparatenkunde*, 1908.

In the design of apparatus, no data as to conductivity values is as valuable to the engineer as those obtained by experiment on similar apparatus under exactly the same working conditions. Such data may, however, be found rare, and if all conditions do not coincide, it may be misleading. Such data, if used, ought to be carefully analyzed into its essential parts. In the lack of such data, resource must

be had to what few values, formulae, and rules are extant on the subject, and by careful piecing together of the same, results of no great engineering inaccuracy may be obtained.

The internal conductivity values for different materials has been subjected to much more accurate determinations than boundary conductivities. The internal conductivity is affected by but one variable temperature; the effect of this is but slight, and need not be taken into account.

Below is given a few values of  $\lambda$  and  $\rho$  in pound, inch, square foot, hour units.

	$\lambda$	$\rho$
Silver.....	3180	0.00031
Copper <sup>1</sup> .....	3020-2660	0.00033-0.00038
Aluminum.....	1000	0.001
Zinc <sup>1</sup> .....	880	0.0011
Tin <sup>1</sup> .....	435	0.0023
Wrought iron.....	450-420	0.0022-0.0024
Cast iron <sup>1</sup> .....	490-310	0.0020-0.0032
Steel <sup>1</sup> .....	320-180	0.0031-0.0056
Lead <sup>1</sup> .....	230	0.0044
Boiler scale <sup>2</sup> .....	22-8.8	0.05-0.11
Marble.....	14.5	0.07
Glass.....	7.2	0.14
Water.....	4	0.25
Glycerin.....	2	0.5
Alcohol.....	1.1	0.9
Ether.....	0.9	1.1
Oil <sup>2</sup> .....	0.8	1.3
Air.....	0.16	6.3
Carbon dioxide ..	0.1	10

Another table given by Lamb and Wilson may be added, although it corroborates certain values given above.

	$\lambda$	$\rho$
Aluminum.....	995-1005	0.001
Brass.....	590-735	0.0017-0.0014
Copper.....	2085-2095	0.00048
Iron.....	480-470	0.0021
Lead.....	245-220	0.0043
Tin.....	445-315	0.00225-0.00415
Zinc.....	880	0.0012
Clay slate.....	7.9	0.12
Granite.....	14.8-16.0	0.065
Marble.....	13.6-16.3	0.07
White dry sand.....	2.7	0.37
Compact sand.....	1.48	0.68
Plaster of Paris.....	3.7	0.26
Pasteboard.....	1.3	0.77
Fir (along grain).....	0.87	1.15
Fir (across grain).....	0.26	3.9
Hair felt.....	0.31-0.42	2.8
Charcoal.....	0.44	2.3
Silicate cotton.....	0.44	2.3
Brown paper.....	0.48	2.1
Pine shavings.....	0.47	2.1
Air (no baffles).....	0.57	1.75
Pure sawdust.....	0.70	1.4
Dry asbestos.....	0.86	1.15
Sand.....	2.15	0.47

Boundary resistivity is very difficult to express in terms of formulae or equations, owing to the number of factors upon which it depends. The resistivity of a water boundary is usually expressed as a function of the mean velocity of the water where that velocity

<sup>1</sup> *Zeitschrift des Vereines deutscher Ingenieure*, 1876.

<sup>2</sup> W. Ernst, *Kaiserliche Akademie des Wissenschaften*, 1902.

is artificially impressed, but where the water is "still," it is not so expressed. The resistivity of denser and more viscous liquids is often referred to the resistivity that water would have under like conditions as a standard. The resistivity of an air boundary is also expressed as a function of the mean air velocity, but the air density has a marked effect, close to the inverse of its cube root. As air is usually at atmospheric pressure, this is not taken into account. The heat resistivity of steam is expressed as a function of its mean initial velocity, although its density has an effect much the same as with air. The steam resistivity is not commonly separated from the water resistivity on the opposite side of the plate, and a favorite formula makes the external resistivity a function simply of the plate or tube area. Finally, to allow for incrustation, heat resistance of the metal, oily surface, etc., a factor is often applied to the external resistance. A more rational way is to add the heat resistance of a known thickness of metal, plus the heat resistance of an assumed thickness of scale and oil onto the external resistance.

Between metal and water, the resistivity  $\zeta = \frac{0.005}{0.3 + \sqrt{v}}$  [ $\gamma = 60 + 200\sqrt{v}$ ] where  $v$  is the velocity of the water in feet per second according to Mollier.<sup>1</sup> When the water is not artificially moved  $\zeta = 0.01$  [ $\gamma = 100$ ], although this will be affected by the temperature difference somewhat, owing to induced currents. For water stirred by mixing devices,  $\zeta = 0.0025 - 0.0013$  [ $\gamma = 400 - 800$ ]. Ser<sup>2</sup> places  $\zeta = \frac{0.002}{\sqrt{v}}$  [ $\gamma = 500\sqrt{v}$ ], but this gives too high a flow of heat.

The external resistance  $z_0$  of a square foot of plate with moving on both sides is the sum of the separate boundary resistivities.

$$z_0 = \frac{0.005}{0.3 + \sqrt{v_1}} + \frac{0.005}{0.3 + \sqrt{v_{11}}}$$

$v_1$  and  $v_{11}$  are the respective velocities of the water on either side.

To obtain  $z$ , the internal resistance of a square foot  $\delta\rho + \delta_1\rho_1$  may be added to  $z_0$ , where  $\delta$  and  $\delta_1$  are the thicknesses of metal and maximum allowable thickness of incrustation in inches, and  $\rho$  and  $\rho_1$  are the respective resistivities. Practical rules given by Hausbrand<sup>3</sup> are that  $z$  may be obtained by multiplying  $z_0$  by 1.5 for copper or brass pipes, by 2.0 for iron pipes, and further this should be multiplied by 1.33 where thick liquids are heated, by 1.33 if the surface is oily, and by 1.66 for very thick liquids.

When water is being evaporated by a hot high boiling liquid moving at a velocity  $v$ ,

$$z_0 = \frac{0.045}{\sqrt{v}} [g_0 = 22\sqrt{v}].$$

When air is heated by hot water or steam pipes, the air boundary resistance is so large that in comparison the other resistances become negligible;

hence,  $\zeta = z_0 = z$ . Joule<sup>4</sup> states that  $\zeta = \frac{0.55}{\sqrt{v}}$  to  $\frac{0.47}{\sqrt{v}}$  [ $\gamma = 1.8\sqrt{v}$  to  $2.1\sqrt{v}$ ], but the latter values were no doubt due to radiation.  $\zeta = \frac{0.5}{\sqrt{v}}$  [ $\gamma = 2\sqrt{v}$ ] is found to hold good for automobile radiators. Mollier<sup>2</sup> proposed  $\zeta = \frac{0.89}{0.36 + \sqrt{v}}$  [ $\gamma = 0.41 + 1.13\sqrt{v}$ ]. For sooty surfaces, Hausbrand recommends  $\zeta = \frac{1.77}{0.73 + \sqrt{v}}$  [ $\gamma = 0.41 + 0.57\sqrt{v}$ ] and where the air is artificially mixed by mixing devices or natural obstructions such as a bank of staggered tubes,  $\zeta = \frac{1.08}{0.45 + v^{\frac{1}{3}}}$  [ $\gamma = 0.41 + 0.92v^{\frac{1}{3}}$ ]. Experiments by E. Josse, of Charlottenburg, on air moving inside of tubes, give  $\zeta = \frac{1.6}{v^{\frac{1}{3}}}$  [ $\gamma = 0.63v^{\frac{1}{3}}$ ] at atmospheric pressure, but with a lower constant for lower pressures.

When air is not artificially moved, the empirical observations of Peclet may advantageously be employed.

The value of  $\gamma$  is found to increase with the temperature difference  $\Delta$ , owing to the higher velocities of induced air currents. For horizontal pipes, where  $\Delta = 20^\circ$ , the value of  $\gamma$  is as follows:

External pipe diameter.....	1/6"	1/2"	1/2"	2/3"	5/6"	1"	1 1/2"	2"	3"	4"
$\gamma$ .....	0.73	0.57	0.52	0.50	0.48	0.47	0.46	0.45	0.44	0.43

For large vertical surfaces,  $\gamma = 0.45 - 0.40$ . When  $\Delta$  is greater than  $20^\circ$ , the value of  $\gamma$  may be obtained by multiplying its value at  $20^\circ$  by a factor.

$\Delta$ .	Factor.	$\Delta$ .	Factor.	$\Delta$ .	Factor.
20	1.00	225	1.69	425	1.97
50	1.22	250	1.73	450	2.00
75	1.31	275	1.77	...	...
100	1.40	300	1.81	...	...
125	1.47	325	1.85	...	...
150	1.53	350	1.88	...	...
175	1.58	375	1.91	...	...
200	1.64	400	1.94	...	...

The resistance of a square foot of pipe where heat travels from air through the pipe to air, where  $v_1$  is the velocity of the air inside of the pipe and  $v_{11}$  is the velocity of the air outside of the pipe, may be found from either of the following equations:

$$Z = \frac{1.6}{v_1^{\frac{1}{3}}} + \frac{0.89}{0.36 + v_{11}^{\frac{1}{2}}} \quad (1)$$

$$Z = \frac{1.6}{v_1^{\frac{1}{3}}} + \frac{1.08}{0.45 + v_{11}^{\frac{1}{2}}} \quad (2)$$

In (1) the air outside moves parallel to the pipe and in (2) the outside air moves perpendicularly against staggered pipes.

From water to air  $\zeta = \frac{1.08}{0.20 + \sqrt{v}}$  [ $\gamma = 0.41 + 2.04\sqrt{v}$ ] where  $v$  is the velocity of the air and the water is broken up into drops as in open-air coolers. When the water is allowed to flow evenly over vertical or

<sup>1</sup> *Zeitschrift des Vereines deutscher Ingenieure*, 1897.

<sup>2</sup> *Physique industrielle*.

<sup>3</sup> Verdampfen, Kondensieren, and Kühlen.

<sup>4</sup> *Philosophical Transactions of the Royal Society*, 1861.

<sup>2</sup> *Zeitschrift des Vereines deutscher Ingenieure*, 1897.

sloping surfaces  $\zeta = \frac{0.75}{0.36 + \sqrt{v}}$  [ $r = 0.41 + 1.33\sqrt{v}$ ].

When the water is still and air passes over it,  $\zeta = \frac{0.89}{0.36 + \sqrt{v}}$  [ $r = 0.41 + 1.13\sqrt{v}$ ].

When the water in moist air is partially condensed by cold tubes, the resistance of a square foot  $z$  may be obtained by adding the separate resistivities which are (1) from water to moving air,  $\zeta = \frac{0.89}{0.36 + \sqrt{v_1}}$ ; (2) through water, assuming  $\delta$  is not greater than  $1/10''$  and  $\rho = 0.25$ , then  $\delta\rho = 0.025$ ; and (3) from metal to the cooling medium,  $\zeta = \frac{0.005}{0.3 + \sqrt{v}}$  for water and  $\zeta = \frac{1.6}{v_{11}^{1/2}}$  for air.

Occasionally, liquids are cooled by flowing over metallic pipes cooled internally by water. The outside air effects cooling both by assisting evaporation and by conduction. For copper or brass pipes,  $z = 0.005$  when the temperature of the liquid to be cooled is above  $60^\circ F.$ ,  $0.006$  when at  $60^\circ F.$  and  $0.007$  when below  $60^\circ F.$  If the metal is iron  $0.001$  should be added; if the liquid is thicker than water  $0.001$ , and if very thick,  $0.002$  should be added. The value of  $\Delta_m$  in this case is based on the cooling water temperatures.

On the resistivity between metal and steam, experimental observations do not lead to satisfactory formulae. Usually the external resistance of a square foot is given. Experiments by Ser<sup>1</sup> on horizontal tubes with water moving outside at a velocity  $v_{11}$  give

$z_o = \frac{0.0022}{v_{11}^{1/2}}$ . Joule,<sup>2</sup> from experiments on a vertical tube of small diameter, found  $z_o = \frac{0.0042}{v_{11}^{1/2}}$ . Nichols

found the ratio of  $z_o$  of a vertical tube to  $z_o$  of a horizontal tube to be  $1^{1/2}$ , but he obtained a higher value for  $z_o$  than Ser. Hagerman<sup>3</sup> found the rate of heat transmission to depend on the temperature of the water and steam. If  $t$  is the steam temperature plus the average water temperature divided by 180, then,

$$z_o = \frac{0.0088}{1 + t} \cdot \frac{0.091}{1 + t + \sqrt{v_{11}}}$$

According to the observations of Hausbrand, if  $v_1$  is the mean initial steam velocity, and  $v_{11}$  is the mean water velocity, then,

$$z_o = \frac{0.0175}{v_1^{1/2} \sqrt{0.023 + v_{11}}}$$

To obtain  $z$  add  $\delta\rho + \delta_{1,\rho_1}$ . For iron pipes, multiply by 1.33.

Where water is pumped through coils surrounded by steam,  $z_o = \frac{0.0096}{\sqrt{0.023 + v_{11}}}$  where  $v_{11}$  is the velocity

of the water;  $z_o = \frac{0.04}{v_1^{1/2}}$  to  $\frac{0.02}{v_1^{1/2}}$  where liquids are warmed up by steam coils without stirring.

These formulae deal with the resistivity from steam through plates to water which is not evaporating. If the water is boiling, other formulae must be used. Mollier obtained  $z_o = 0.0014$ , but where rigorous precautions were employed to completely exclude traces of air from the steam,  $z_o = 0.0007$ . Mollier assumed that the resistivity from metal to water is the same as from metal to steam,  $\zeta_1 = \zeta_{11} = 0.00035$ . The increased resistance is believed to be due to a film of air. The resistance of a square foot of an air film would be  $0.001$  on this assumption, and since the heat resistivity of air is 6.25 the film thickness would be  $0.0002''$ .

From experiments on evaporation, Jellinck<sup>1</sup> proposed the following formula, expressing the resistivity in terms of  $c$  the circumference of the steam pipe in feet, and  $l$  its length in feet, or  $A$  the area in square feet.

$$z_o = 0.00045c^{1/2} l^{1/2} = 0.00045 \sqrt{A} [g_o = \frac{2200}{\sqrt{A}}]$$

Actually,  $l$  does not vary as  $l^{1/2}$  but as  $l^{1/3}$  to  $l^{1/4}$ . Where  $l$  is very large or small this formula does not apply. To allow for incrustation, etc., the following factors of safety are recommended by Hausbrand.<sup>2</sup>

Ratios of $z/z_o$ .	Water.	10-25% solid.	Viscous liquids.
Copper tubes.....	1.5	2.0	2.5
Wrought iron tubes.....	2.0	2.5	3.0
Cast iron tubes.....	2.5	3.3	4.0
Lead pipes.....	3.0	4.0	4.5

For thick viscous liquids;

- $z = 0.075-0.0065$  long heating coils.
- $= 0.0060-0.0055$  short heating coils.
- $= 0.0050$  thin heating coils.
- $0.0070-0.0080$  vertical systems of tubes with steam outside.

In vacuum evaporators, the resistance may become 1.2 to 1.5 times that given above.

In the case of steam-jacketed kettles, if  $\Delta$  is the temperature difference between the evaporating liquid and the initial steam temperature, then  $\Delta_m$  should be chosen less than  $\Delta$  as is given in the following table:

Diameter of kettle.	$z_o$ .	$\Delta_m/\Delta$ .
Up to $2\frac{1}{2}'$ .....	0.0027	0.85
$2\frac{1}{2}'-5'$ .....	0.0029	0.80
$5'-7\frac{1}{2}'$ .....	0.0031	0.75
$7\frac{1}{2}'-10'$ .....	0.0033	0.70
$10'$ and over.....	0.0035	0.65

For iron,  $\frac{z}{z_o} = 1.33$ . If there are two steam openings this should be multiplied by  $3/4$ , for four steam openings multiply by  $2/3$ , and if a stirring apparatus is used  $1/2$  should be used to multiply with.

For vacuum steam-jacketed kettles:

- $z = 0.0040-0.0048$  water.
- $= 0.0048-0.0055$  thin liquid.
- $= 0.0055-0.0100$  thick liquid.

Where multiple-effect evaporators working on liquors presenting no special difficulties, ordinary depths

<sup>1</sup> *Physique industrielle.*

<sup>2</sup> *Philosophical Transactions of the Royal Society, 1861.*

<sup>3</sup> *Engineering, 1875.*

<sup>4</sup> *Proceedings of the Institute of Civil Engineers, 1844.*

<sup>1</sup> *Zeitschrift des Vereines für Rubenzucker Industrie, 1894.*

<sup>2</sup> *Verdampfen, Kondensieren, and Kühlen, 1909.*

of liquor being maintained and heated by brass tubes of three feet or over in length, the values of

$$\Delta g = \frac{Q}{TA}$$

are as follows:

Single effect.....	14000-16000	B. t. u. per sq. ft. per hour
Double effect.....	6000- 7500	B. t. u. per sq. ft. per hour
Triple effect.....	4000- 5200	B. t. u. per sq. ft. per hour
Quadruple effect.....	3750- 4400	B. t. u. per sq. ft. per hour

Where the depth is lower than usual these figures are 10 per cent. higher, wide horizontal tubes 10 per cent. more, narrow, horizontal tubes 15 per cent. more, iron horizontal tubes 10-15 per cent. less, and for evaporators of the spraying or showering types, 10 per cent. more.

### THE UTILITY OF THE METALLOGRAPHIC MICROSCOPE.

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This article will be confined entirely to the metallographic microscope, without touching upon its use in the immense fields of geology and botany, where the examination of rock sections and vegetable products has a direct utility in engineering. Here, however, the methods are somewhat different than in metallography, since transparent sections and transmitted light (often polarized light) are used. Also, it is not purposed to dwell especially upon the technique of microscopic metallography. The methods employed are presented in good form in several books and numerous articles devoted to the subject. Again no detailed or elaborated discussion of any complex phase of the subject will be taken up. Rather, concrete examples will be brought forward illustrating what metallography has done for us, with the hope that the discussion will open suggestions of its possibilities in individual fields of endeavor.

Metallography is but a step towards the rationalizing of our study of metals and alloys. The earlier work in the study of these materials was naturally along the lines of their possible uses; it consisted of tests of these various properties and a classification according to their several fields of service. A vast accumulation of data of this kind led to the adoption of a rather empirical correlation of composition and quality. Chemical analysis later came in to give us a more exact correlation by the elimination of one big uncertainty, the composition. But chemical analysis falls short of giving us sufficient information regarding our material. Substances owe their properties solely to their make-up, or structure. Composition is but one of the determining factors; chemical analysis therefore fixes but one of the variables. The relation of these chemical elements or the make-up of the material is most vitally dependent on its history. Temperature and pressure conditions, or the proximity of disturbing influences, are of importance. In steels, for example, we consider the effects of heat and mechanical treatment; that is, for identical analyses, a steel annealed at 1200° C. would be weaker than one annealed at 800° C.; one quenched at 800° C. would be harder than the same material slowly cooled from

that temperature, while cast material is weaker than that which has had subsequent rolling or forging.

Structure or make-up is therefore the first step in the interpretation of the properties of materials. An essential in this determination is the composition; chemical analysis is therefore a necessity, and it must be understood clearly that metallographic study is only an auxiliary to the other methods of test. Two lines of attack are open: by the use of the pyrometer and by the use of the microscope. The latter was developed much the earlier, having its beginning in the work of Dr. Sorbly in England in 1864 and the later independent investigations of Martens in Germany in 1875. However, the real development has come about within the last ten years; this is due to the rapid progress in physical chemistry, particularly in a better understanding of the theory of solutions. In the earlier work it was possible to develop the structure of the material under examination, but the full measure of the value of the work was missed through the inability to interpret the developments; again systematic research was handicapped by this lack of knowledge. To-day we have a rational interpretation of the results of solidification of molten mixtures and a consequent explanation, more or less developed, of the variations in the structure of alloys occasioned by changes of composition or treatment.

It will be impossible in this paper to dwell upon the use of the pyrometer in the study of the structure of alloys. Cooling curves obtained by its use undoubtedly furnish the most rational basis for the theoretical deductions of the results to be expected on solidification of the melt. But outside of its very direct practical bearing in certain instances, notably in the determination of the critical points of tool steels, the pyrometer may be said in general to be of more scientific interest. The microscope, on the other hand, has a more direct application in the industrial laboratory since with it one is able to make an examination of any desired material as it is used or as it exists, and to interpret from the structure, backed by experience, the condition of the material and its suitability for the purpose in view.

Microscopic examination of metals and alloys is passing through the same cycle that the chemical analysis of iron and steel did some years ago. We have the enthusiast, who would claim for the microscope the power to usurp the functions of other testing methods; and the skeptic, who treats it as a toy and scoffs at any suggestion of utility outside of the laboratory of the scientific investigator. The chief argument of the latter is that the field of view of the microscope is only an extremely small part of a very small sample cut from a large mass of material and it can disclose, therefore, only local conditions. Such argument is hardly tenable; it would apply with equal force to the one-gram sample, the chemical analysis of which controls the 60-ton heat of a steel furnace; or to the purchase of a car-load of coal or a boat-load of ore on determinations of heating value, composition, etc., made on equally small units; or to our methods of design of structures from material whose