159. A Note on the Gnomonic Projection
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of each other in the plane of s from P and P' respectively are enantiomorphous —which is obvious (cf. 154, p. 88). If we use the gnomonic projection instead of the stereographic, we have: If $\alpha(F)$ denote the harmonic homologue of any figure F, A being the centre of homology and the polar of A, with respect to a given circle the axis of homology, then $\pi(F)$ and $\pi'(F')$ are enantiomorphous, where $\alpha(F')=F$, $\beta(A)$ is at infinity, $\beta(P)=P'$, and AB passes through the centre of the given circle (cf. Note 157).

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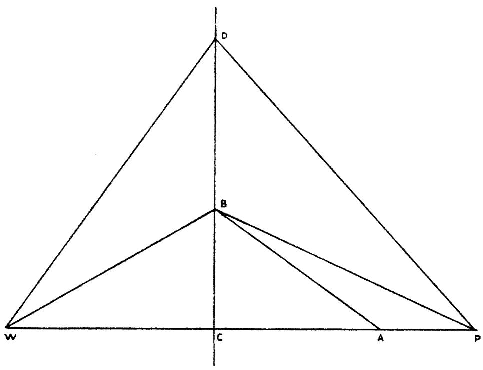
159. [P. 3. b. α.] *A note on the gnomonic projection.*

Let B, C, D, P, ... be the projections from O on the tangent plane at A of the points B', C', D', P', ... on a sphere whose centre is O. (B may be conveniently marked with a × or a o according as the angle BOA is acute or obtuse.) Draw $PACW$ perpendicular to BD meeting BD in C; let $WC^2=AC^2+AO^2$ and $CA \cdot AP=AO^2$. Then the angle $BWD=B'OD'=$ the angle between the great circles $B'P'$, $D'P'$, since $B'P'$, $D'P'$ are quadrants, and any point on BD is equidistant from O and W. These facts enable us to measure the angle and sides of any spherical triangle whose gnomonic projection is given. They also enable us to solve graphically any spherical triangle; for the gnomonic projection of any spherical triangle on the tangent plane at a vertex can be readily found when the three sides, or the two sides meeting at that vertex and any angle are given.



Moreover the projection enables us to prove the usual formulae of spherical trigonometry. For example: since

$$\sin BAC=\frac{BC}{BW}\div\frac{BA}{BW};\ \cos BAC=\frac{AC}{AO}\div\frac{AB}{AO};\ \tan BAC=\frac{CB}{CW}\div\frac{CA}{CW};$$

we have the following formulae for a spherical triangle right-angled at C: $\sin A=\sin a \cdot \operatorname{cosec} c$; $\cos A=\tan b \cdot \cot c$; $\tan A=\tan a \cdot \operatorname{cosec} b$.

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160. [L¹. 3. a.] *On the Equation to the axes of the general conic.*

The points on the ellipse $x^2/a^2+y^2/b^2=1$ the normals at which cointersect in a given point (ξ, η) are determined as the intersections of the ellipse with a definite rectangular hyperbola $a^2\xi/x-b^2\eta/y=a^2-b^2$. When $\xi=0$, $\eta=0$ this R.H. degenerates into the two axes of the ellipse.

Similarly the points on the general conic the normals at which cointersect in a given point (ξ, η) lie on the R.H.

$$(ax+hy+g)/(x-\xi)=(hx+by+f)/(y-\eta).$$

Hence making (ξ, η) the centre of the curve, the equation to the axes of the conic is

$$(ax+hy+g)(y-F/C)=(hx+by+f)(x-G/C).$$

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