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and so on. I do not remember seeing a proof, but I think $2^{n-1} - 1$ is never divisible by n^2 . Are there other such cases?

If, on the other hand, n be given, there are $n - 2$ distinct values (besides 1) of m_1 for which

$$(m_1 + kn^2)^{n-1}$$

is divisible by n^2 for all integral values of k . Any one of them may be thus determined. Let m_1 be any integer in the limits

$$1 < m_1 < n.$$

Then $m_1^{n-1} = 1 + \mu n + \mu' n^2$,

where $0 > \mu > n - 1$.

Let $m_1^{n-2} = \lambda + \lambda' n$,

where $1 < \lambda < n$,

then $(m_1 + \kappa n)^{n-1} = m_1^{n-1} + m_1^{n-2} \cdot \kappa n(n-1) + \text{a multiple of } n^2$

$$= 1 + (\lambda - \kappa \mu) n + \text{a multiple of } n^2.$$

Hence $(m_1 + \kappa n)^{n-1}$ is divisible by n^2 , if and only if $\lambda - \kappa \mu$ is divisible by n , i.e. if

$$\lambda \mu^{n-2} - \kappa \mu^{n-1} \text{ is divisible by } n,$$

or if $\lambda \mu^{n-2} - \kappa$ is divisible by n .

Thus if κ is the remainder in dividing $\lambda \mu^{n-2}$ by n and $m_2 = m_1 + \kappa n$, then $(m_2 + kn^2)^{n-1}$ is divisible by n^2 , and there are therefore (excluding units) $n - 2$ distinct values of m_2 corresponding to the different values of m_1 .

It is obvious that if m_2 satisfy this condition, so do $m_2^2, m_2^3, \dots, m_2^{n-2}$. But if M_1 be a primitive root of n [i.e. if $n - 1$ be the smallest value of m for which $M_1^m - 1$ is divisible by n], and M_2 the corresponding value of m_2 , then $M_1, M_1^2, \dots, M_1^{n-2}$ on division by n must leave the remainders $2, 3, \dots, (n - 1)$ in some order: so too must $M_2, M_2^2, \dots, M_2^{n-2}$. Hence, we may take for our set of distinct solutions the remainders on dividing $M_3, M_2^2, \dots, M_2^{n-2}$ by n^2 .

Just as m_2 (or M_2) was determined from m_1 (or M_1), so may m_3 (or M_3) be determined from m_2 (or M_2), and so on in succession. W. E. H.

254. [v. 1. a.] "Approximately equal to."

I should like to suggest the use of a broken sign of equality \neg for "approximately equal to."

Thus, I should write

$$\tan \theta = \frac{5}{1560} \neg \frac{1}{36}, \text{ whence } \theta \neg 2^\circ.$$

$$x = \sqrt{2} - 1$$

$$\neg \cdot 41.$$

The sign is a long and short \neg , with a slight tick at the end of the short to avoid confusion with a hastily written $=$. C. S. J.

255. [K. 8.] Let A, B, C, D be the vertices of a quadrilateral; $EF G$ its diagonal triangle. L, L', M, M', N, N' the mid points of AC, BD, AB, DC, DA, CB respectively; efg the median triangle of the triangle $EF G$.

It is a defect in all our text-books to exhibit the collinearity of $LL'e$ and to ignore the collinearities of $MM'g$ and $NN'f$.

The four points A, B, C, D give rise to three quadrilaterals, the pairs of diagonals being (AC, BD) ; (AB, CD) ; (AD, BC) .

Students are apt to think that LL' possesses some special property not shared by MM', NN' . W. GALLATLY.

256. [K. 6. a; 13. c.] On the area of a triangle, the equations of whose sides are given.

Let the sides be $a_1x + b_1y + c_1 = 0, \dots \dots \dots (1)$

$$a_2x + b_2y + c_2 = 0, \dots \dots \dots (2)$$

$$a_3x + b_3y + c_3 = 0, \dots \dots \dots (3)$$

and let (1) contain the vertices $(x_2, y_2), (x_3, y_3)$, and similarly for the others.