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REACTANCE.

BY CHARLES PROTEUS STEINMETZ AND FREDERICK BEDELL.

The term "reactance,[1]" which has been used and advocated by the writers and others, and which has been officially adopted at our suggestion by the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS at its Philadelphia meeting, is one which assumes particular importance now that the term "inductance" is defined as synonymous with the "coefficient of self-induction," usually symbolized by the letter "*L*." Reactance is similar in many respects to resistance, but the electromotive force used in overcoming reactance consumes no power, for it is at right angles to the current.

The impressed electromotive force in an alternating current circuit may be divided into two components: First, the *power electromotive force* in the direction of the current, and, second, the *reactive electromotive force* in quadrature with the current to overcome the reactance. The reactive electromotive force is the product of the current and the reactance. *The reactance is, accordingly, equal to the component of the impressed electromotive force at right angles to the current, divided by the current.* Reactance is measured in ohms.

The reactive electromotive force in the circuit may be due to self or mutual induction, to capacity or to some outside counter electromotive force produced by a motor or other device. In general, in any alternating current circuit,

1. This term was first suggested by M. Hospitalier (see *L' Industrie Electrique*, May 10, 1893,) and was proposed officially (see *Bulletin*, June, 1893,) by the committee appointed by the Société Internationale des Electriciens to consider the Congress proposals of this INSTITUTE. (See TRANSACTIONS, vol., x., p. 413.) We take pleasure in expressing our appreciation of the praiseworthy efforts of Prof. Hospitalier and the Société Internationale in their advocacy of this and other conventions of nomenclature.

$$\text{Impressed E. M. F.} = \sqrt{\text{Power E. M. F.}^2 + \text{Reactive E. M. F.}^2};$$

that is, the impressed electromotive force is the vector sum of the electromotive force which transmits power and the reactive electromotive force.

Reactance tends to cause a phase difference between current and electromotive force. If θ represents this angle of phase, we have the relation,

$$\tan\theta = -\frac{\text{reactance}}{\text{resistance}}.$$

When θ is negative, the current lags behind the electromotive force; when θ is positive, the current is in advance of the electromotive force. The expression for the instantaneous value of the current may be written.

$$i = I \sin(\omega t + \theta),$$

or

$$i = \frac{\text{Impressed E. M. F.}}{\text{Impedance}} \sin\left\{\omega t - \text{arc tan} \frac{\text{reactance}}{\text{resistance}}\right\}.$$

A few illustrations of particular cases will make the use of the term reactance more clear. For simplicity in these illustrative examples, we will consider that no iron is embraced by the circuit.

Circuits Containing Resistance and Inductance.—In a simple circuit containing resistance and non-ferric inductance, the reactance is equal to $L\omega$; that is, it is 2π times the product of the inductance and frequency. The impedance being the vector sum of the resistance and reactance, is in this case

$$\text{Impedance} = \sqrt{R^2 + L^2\omega^2},$$

where $\omega = 2\pi \times$ frequency. In this case all the power is used in overcoming resistance, and the power electromotive force is equal to the ohmic electromotive force, RI. The reactive electromotive force is equal to the inductive electromotive force $L\omega I$; hence

$$\text{Impressed E. M. F.} = \sqrt{\text{Power E. M. F.}^2 + \text{Reactive E. M. F.}^2};$$

or,

$$\text{Impressed E. M. F.} = \sqrt{\text{Ohmic E. M. F.}^2 + \text{Inductive E. M. F.}^2}.$$

The impressed electromotive force in this case is the vector sum of the electromotive forces necessary to overcome resistance and inductance.

A consideration of Figs. 1 and 2 (in which positive direction is counter clockwise) will show that the reactance and reactive electromotive force are *positive* in the case of a circuit containing inductance. The impressed electromotive force which is the sum of the two, is, therefore, in advance of the current, which is in the same direction as the power or ohmic electromotive force. The current is indicated by a closed arrow in the figures. The same, however, may be otherwise expressed if we take the impressed electromotive force as our direction of reference, by saying that the *current lags behind* the electromotive force by the angle θ, which angle is negative, therefore, for circuits with inductance according to the relation

$$\tan\theta = -\frac{\text{reactance}}{\text{resistance}} = -\frac{L\,\omega}{R}.$$

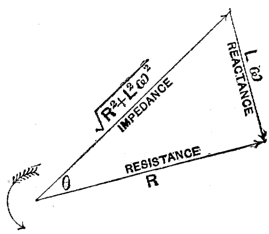

Fig. 1.

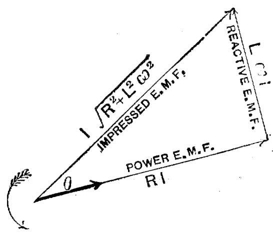

Fig. 2.

The value of the current at any instant is given by the equation

$$i = \frac{E}{\sqrt{R^2 + L^2\,\omega^2}} \sin\left(\omega\, t - \text{arc tan}\,\frac{L\,\omega}{R}\right).$$

Circuits Containing Resistance and Capacity.—In such circuits the reactance is equal to $-\frac{1}{C\,\omega}$, where $C =$ capacity, and $\omega = 2\,\pi \times$ frequency, that is, it is negative. The reactive electromotive force is also negative, being equal to $-\frac{I}{C\,\omega}$. This negative reactance gives a positive value to the angle θ, and the current is accordingly in advance of the electromotive force. We have then

$$\text{Impedance} = \sqrt{R^2 + \frac{1}{C^2\,\omega^2}},$$

and

$$\tan\theta = -\frac{\text{reactance}}{\text{resistance}} = +\frac{1}{C\,R\,\omega},$$

similar to the corresponding equations above. The instantaneous value of the current is given by the equation

$$i = \frac{E}{\sqrt{R^2 + \frac{1}{C^2 \omega^2}}} \sin \left(\omega t + \text{arc} \tan \frac{1}{C R \omega} \right).$$

These relations are shown in Figs. 3 and 4. They assume the absence of dielectric hysteresis.

Circuits Containing Resistance, Inductance and Capacity.—In a circuit containing inductance and capacity the reactive electromotive force is the sum of the inductive electromotive force, and the condenser electromotive force; where the inductance is non-ferric and dielectric hysteresis absent, this is, $L \omega I - \frac{I}{C \omega}$. The reactance is similarly the sum of two terms, $L \omega - \frac{1}{C \omega}$;

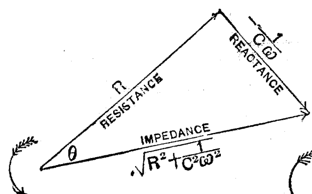

FIG. 3.

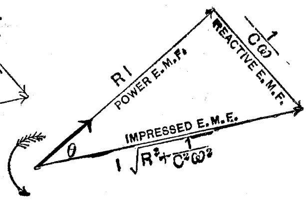

FIG. 4.

thus, the reactance is positive or negative, according to whether the inductive electromotive force is greater or less than the condenser electromotive force. Whether the current is behind or ahead of the impressed electromotive force depends upon the same condition, for

$$\tan \theta = - \frac{\text{reactance}}{\text{resistance}} = \frac{1}{C R \omega} - \frac{L \omega}{R}.$$

The value of the current at any instant is

$$i = \frac{E}{\sqrt{R^2 + \left(L \omega - \frac{1}{C \omega} \right)^2}} \sin \left\{ \omega t + \text{arc} \tan \left(\frac{1}{C R \omega} - \frac{L \omega}{R} \right). \right\}$$

The diagrams in Figs. 5 and 6 illustrate the case of a single circuit containing resistance, inductance and capacity. In the case here represented, the condenser electromotive force is greater than the inductive electromotive force, and the current is, therefore, in advance.

Circuits Containing Mutual Induction.—To further exemplify the use of the term reactance, let us consider a circuit containing a transformer. Besides the electromotive forces already discussed, we must consider the back electromotive force due to the influence of the secondary upon the primary circuit. This may be resolved into two components, one in the same direction as the current, and the other in quadrature with it, which will form part of the power electromotive and reactive electromotive forces, respectively. The electromotive force introduced by a motor is treated in the same way.

These illustrations will suffice to show the method by which any electromotive force in a circuit is resolved into two components, one in the direction of the current, which, therefore, transmits power, and the other at right angles to the current, which represents no power, but simply overcomes the reactance.

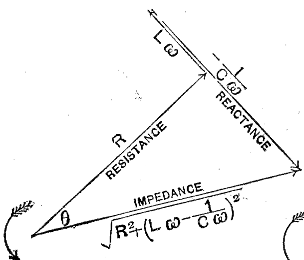

Fig. 5.

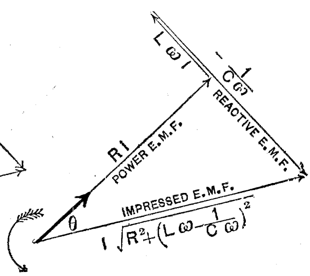

Fig. 6.

It is to be borne in mind that the terms here given may be used in all alternating current circuits, whether the current is harmonic or not; for, when the current is not strictly harmonic, we may consider it equivalent to an harmonic current. The *equivalent* harmonic current and equivalent harmonic electromotive force have the same square root of mean square values as the actual current electromotive force, and have such relative phase positions that the same power is transmitted.

APPENDIX.

The Definition of Reactance.

[Communicated by the authors, October 31, 1894.]

At the Philadelphia meeting of the American Institute of Electrical Engineers, May, 1894, the name "reactance" was officially adopted by the Institute and defined in accordance with a paper on Reactance communicated by us and reprinted in most of the electrical periodicals. The action of the INSTITUTE was unanimous and its definition has met with general approval. Objection to it, however, has been raised in an article[1] recently communicated to a French periodical. Since we cannot agree with the arguments forwarded in this article, it will be in place for us to state our views thereon.

The definition of reactance adopted by the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS is:

"Reactance is the component of electromotive force at right angles to the current, divided by the current."

By this definition the term reactance is the quotient of the reactive E. M. F. divided by the current, where the reactive E. M. F. includes all wattless E. M. F.'s, whether due to inductance, capacity, polarization or counter E. M. F. of any kind, as of synchronous motors, and excludes all energy components, as would be introduced by motors, transformers, hysteresis, etc.

In objecting to this definition, the writer referred to takes a position essentially as follows:

(1.) The term reactance shall include the effects of self-induction and capacity only.

(2.) It should always be defined by the equation

$$I = \frac{E}{\sqrt{R^2 + K^2}};$$

Where: I = current;
R = ohmic resistance;
E = electromotive force;
K = reactance.

(3.) For harmonic currents

$$K = \omega L - \frac{I}{C\,\omega};$$

Where: L = self-inductance;
C = capacity;

$$\omega = \frac{2\,\pi}{T} = \text{pulsation.}$$

(4.) The term "reactance" has a right to exist only because it is a constant of the circuit. Defined, however, as the quadrature component of E. M. F. divided by the current, it is the complex resultant of different reactions and not a constant of the circuit. All terms in "ance" should denote constants of the circuit, and whenever used in a generalized meaning, an additional term should be added, as "apparent reactance."

We may state that when investigating the question of properly defining the term "reactance," we have fully considered the

1. "A propos de la Reactance," by Professor André Blondel. *L'Industrie Electrique,* Sept. 25, 1894.

position taken by this writer. We have, however, come to the conclusion that his definition is not tenable, but is contradictory, for the following reasons:

(1.) Neither in the one nor in the other definition is "reactance" a constant of the circuit, except in circuits containing no iron. In reality, circuits nearly always contain iron, and in such circuits the reactance can be considered as approximately constant only in a very limited range. When extended over a greater range of E. M. F. or of current, the self-inductance, and thus the reactance, varies. The same applies to most of the other quantities ending in "ance," as impedance, reluctance, and permeance, and thus the statement that quantities in "ance" should be constants of the circuit, is against the adopted practice and not fulfilled by either definition.

(2.) Where iron is present, the statement for harmonic currents

$$K = \omega\, L - \frac{I}{C\,\omega}$$

contradicts the definition of K by the equation

$$I = \frac{E}{\sqrt{R^2 + K^2}},$$

R being taken as the ohmic resistance and K as reactance. This is due to the presence of hysteresis. These relations do hold in the absence of iron, and such cases were taken by us in our paper to illustrate the definition for simple cases; the fundamental definition, however, should in our opinion be sufficiently general to include circuits with iron. Consider an harmonic current flowing in a circuit embracing iron; for simplicity assume capacity absent. The last equation would not give any direct relation between the inductance and reactance, such as would be obtained from the equation just preceding, but would make reactance still a more complex quantity, by including therein not only the effect of self-induction but that of hysteresis as well. Furthermore, it would lead to the result that the reactive E. M. F. is not wattless, but includes an energy component, an idea quite foreign to the term.

These considerations obliged us to discard any indirect definition of reactance by means of the term inductance, and to adopt the more direct definition analogous to the definition of resistance.

The definition of resistance is Ohm's law:

$$R = \frac{E}{I},$$

where E and I represent an unvarying E. M. F. and current, respectively.

Considering E and I as an harmonic electromotive force and an harmonic current—or in the case of alternating quantities which are not simple harmonics, as their equivalent harmonic values, that is, values of equal square root of mean square value and equal power—the above equation gives the definition of impedance:

$$\text{Impedance} = \frac{E}{I}.$$

Our proposed definition for reactance is analogously written:

$$\text{Reactance} = \frac{\text{reactive } E}{I}.$$

Further, we may define the apparent[1] resistance of the circuit by the expression

$$\text{Apparent resistance} = \frac{\text{power } E}{I}.$$

In the absence of expenditure of energy outside the electric conductor, this quantity coincides with the true ohmic resistance.

These quantities are thus defined directly, and in a uniform manner. How far these quantities are constants of the circuit depends upon the circumstances; in any case reactance and impedance depend upon the frequency.

The use of "equivalent" values for quantities is employed in the paper on "The Law of Hysteresis, III." In the discussion of this paper at the Philadelphia meeting the significance of these values is pointed out.

The phenomena taking place in an alternating current circuit cannot fully be represented by the terms:—ohmic resistance, inductance, and capacity, but a further term has to be introduced, representing the losses of energy outside of the electric conductor, as hysteresis, etc., and the most satisfactory way to do this appears to us to be the generalization of the term "resistance" and to denote by it "apparent resistance," or by some similar term.

Wherever the reactance is generalized to include active counter E. M. F.'s., it may well be distinguished by the denotation "equivalent reactance."

We would emphasize the utility of *two rectangular components*, whether E. M. F.'s or currents are resolved. Reactance should always, in our opinion, be associated with that which represents no expenditure of energy, the reactive E. M. F. being at right angles to the current and the reactive current (the wattless current) being at right angles to the E. M. F. Whether we resolve currents or E. M. F.'s depends upon the problem in hand; each method has its advantages. If we resolve the current, we may write:

$$(\text{Current})^2 = (\text{power current})^2 + (\text{reactive current}).^2$$

Divided by E, this gives:

$$(\text{Admittance})^2 = (\text{conductance})^2 + (\text{susceptance})^2,$$

1. The term apparent resistance is here used in contradistinction to the true ohmic resistance. Whether the word apparent is the best one for generalizing the term resistance is an open question; we so use the word in the present communication.

for a simple case; in general, for conductance and susceptance we should write apparent conductance and apparent susceptance. Admittance, conductance and susceptance are thus used as the inverse correspondents of impedance, resistance and reactance, and may be added as vector quantities. Many alternating current problems are much simplified by this treatment. It is important, however, to employ components which are at right angles to each other, and for this reason the definition of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS seems preferable. It is in this point that the definition is fundamentally different from that of the French writer already referred to. In the absence of hysteresis losses, the definitions would be the same, applying the term equivalent reactance to the case where counter E. M. F.'s other than those due to capacity and self-induction, are present. To conclude, then, we may say that in the absence of iron we may define reactance in terms of inductance and capacity, as this writer has done, and as has been done by us in the illustrative examples in our paper; the fundamental definition, however, should, in our opinion, remain in the general form adopted by the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

In a note to our original paper, we have called attention to the first suggestion of the term reactance by M. Hospitalier in May, 1893, and the recommendation of the committee appointed by the Société Internationale des Electriciens to consider the programme for the Chicago Congress, 1893. The term is a happy one; it is international, and uniformity in its use is to be desired.

In our opinion the best definition for "reactance" is that adopted by the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS. In this, as in all matters, there is however room for difference of opinion. The reasons for thus defining the term have not before been published, but we believe that when they are duly considered, the action of the INSTITUTE will meet with international approval.

October 31, 1894.

[COMMUNICATED BY PROF. H. J. RYAN, NOVEMBER 21ST, 1894.]

I like the definition of "reactance" as put forth by the authors of this paper. Had we learned to use alternate currents first, this relation between the constant properties of a circuit, the current and E. M. F. would first have been understood instead of Ohm's law:

$$\text{Impedance} = \frac{E}{I}.$$

Experience would next have taught us that this impedance relation is made up of two fundamental component relations: the one is a power relation, and the other, a wattless relation. Be-