INTERFERENCE IN THIN FILMS — A GRAPHICAL TREATMENT.

BY E. R. DREW.

WHEN light falls upon a thin film of a transparent substance, interference fringes are observed in the portions of light reflected from the two surfaces. These fringes, under given conditions, appear to be localized at a particular place, so that they are not seen distinctly with a telescope unless it is focused upon a point at a particular distance from the film. This feature of the phenomenon of interference due to thin films has seemed rather difficult of explanation.

Michelson [1] has given a mathematical solution of the problem for the cases which arise in the form of interferometer devised by him, in which the light is always incident normally or nearly so; and Feussner [2] has given an elaborate treatment of the general case of light incident at any angle upon a film whose plane surfaces make a small angle with each other. More recently H. A. Pocklington [3] has proposed a method of analysis which is much simpler and clearer than that of Feussner.

It has seemed to the writer that it might be an aid to clearness of thought with regard to the whole phenomenon if it could be treated in a graphical way, by which the wave-surfaces could be constructed, and the interference effects obtained by the intersections of these surfaces.

The method is often used in making clear some simple cases of interference, as, for instance, the effects which are seen when a distant arc light is viewed through two pinholes close together in a card which is held just in front of the eye. In this case each pinhole becomes effectively a source of light, sending out disturbances

[1] Winkelmann, Handbuch der Physik, II., 1, p. 546.
[2] Phil. Mag. (5), 13, p. 236, 1882.
[3] Proc. Camb. Phil. Soc., XI., 2, p. 105, April, 1901.

in the form of spherical waves into the region behind the card. The waves in the two sets are similar in every respect, having come from the same actual source, and so are able to produce definite interference effects.

In Fig. 1, A and B represent these two effective sources, and the circular arcs represent a series of corresponding wave-fronts of the two systems of waves. Any point in which a wave-front of system A intersects one of system B is evidently a point of maximum disturbance; and as the wave-fronts move outward, all points on the loci of these intersections will be points of maximum disturbance continuously. These loci are obviously portions of a system of hyperbolas, having A and B as foci; but within the region considered their curvature is so small that they may be well enough represented by straight lines. If a screen be placed within this region, so that the light passing through the two pinholes falls upon it, there will be a maximum of intensity at each point where it is cut by one of these lines. In this case the fringes have no definite focus.

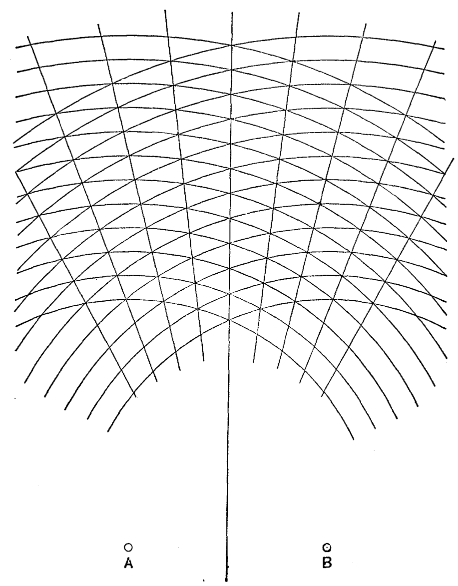

Fig. 1.

Consider next a thin film of air enclosed between two glass plates, with the light from a sodium burner falling upon the film at any angle, and being reflected from its two surfaces. The figures are plane sections perpendicular to the surfaces of the film, and passing through the source of light and the eye of the observer. The desired simplicity of treatment can only be attained by considering a somewhat ideal case, in which the effect due to refraction in the upper glass plate is neglected. But the two portions of light which come by reflection from the two surfaces of the film are refracted by this plate in almost identical fashion, so that any relations which exist between them will not be appreciably changed thereby.

In Fig. 2, the two surfaces of the film are taken parallel. A is a particular point in the sodium flame which serves as the source of light, A' its image formed by reflection at the upper surface of the film, A'' that formed at the lower surface. The two reflected portions of light will then diverge from these two points as virtual sources. Finding the loci of the points of maximum disturbance as in Fig. 1, a_4 may represent the locus of points whose difference in distance from A' and A'' is four wave-lengths, a_3 the similar locus for a difference in path of three wave-lengths. In this and the following figures, the loci are constructed by locating two points on a chosen hyperbola, and drawing a straight line through them.

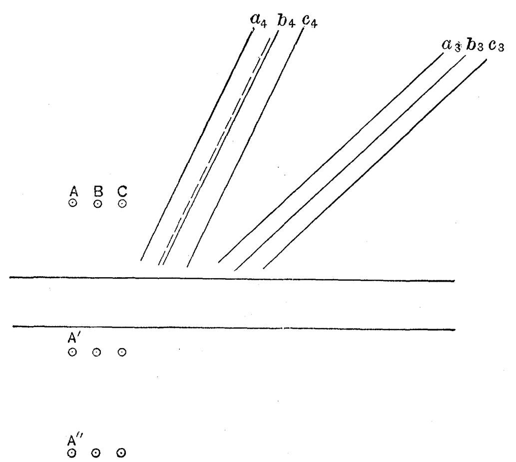

Fig. 2.

This construction is next carried out for two other points, B and C, on the luminous source, giving the loci b_4, c_4, and b_3, c_3. The three lines in each set are evidently parallel. There is actually a continuous series of luminous points across the source where it is intersected by the plane of the figure, and the loci for all of these points will similarly be parallel in each set. But for some point on the source, a locus of maximum disturbance will lie midway between a_4 and a_3, which will be the locus of minimum disturbance for the point A on the source. This overlapping of maxima and minima on a screen placed in the path of the reflected light will evidently prevent the formation of any definite interference effect. But if a portion of this reflected light enter the eye, the observer will see a maximum illumination coming apparently along the path a_4 and all paths parallel to it, and therefore coming apparently from the point where these paths intersect, at infinity. Looking along a direction midway between a_4 and a_3, he will similarly see a minimum effect coming along a set of parallel paths. The interference fringes will then be clearly seen by the eye, or in a telescope, only when it is focused for parallel light.

It is of course to be noted that definite interference cannot occur between portions of light coming from two different points, A and B, on the source, but only between different portions of the light which originally came from a single point on the source.

There are other portions of the light from A which will emerge from the film after multiple reflection within it, and interference may occur among any two of these portions. The locus for one such case is shown by the dotted line in the figure; and it is evident that they will all be parallel to those already found.

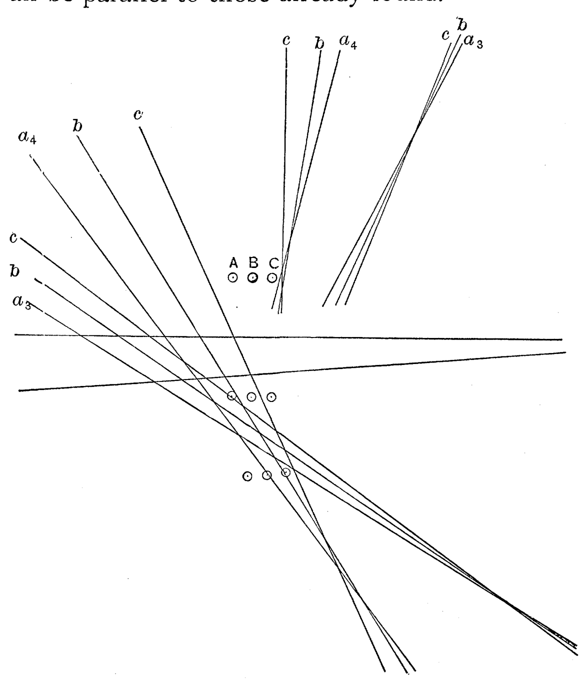

Fig. 3.

In Fig. 3 the same construction is applied to a wedge-shaped film. The two lines a_4 here represent two portions of the same hyperbola, in one case the light being reflected toward the angle of the wedge, in the other away from it. The two loci a and b now intersect at various points near the film. This means physically that at such a point of intersection there is a maximum of intensity in the illumination due to the point A on the source, and also in that due to B. It means, also, that the eye will see a maximum illumination coming along the two directions a and b, which will consequently seem to be localized at their intersection, and so a sharply defined bright fringe will be seen in a telescope focused upon this point.

The construction shows that any three loci do not in general intersect in a point. It is quite suggestive of the similar construction for the wave-normals of light obliquely refracted at a plane surface.

The distance of the fringes from the film varies with the angle of incidence, becoming zero, and changing sign, with it. The distance also varies with the thickness of the film; for if the film, imagined extended beyond the vertex, be moved toward the left, keeping the angle of incidence fixed, the fringes will pass through the film as the vertex, where the thickness is zero, passes the point of incidence. It is also evident that the distance will vary inversely with the angle of the film, for the distance becomes infinite when the angle becomes zero.

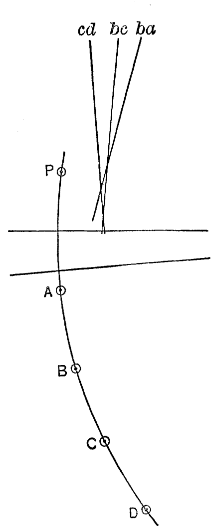

Fig. 4.

Fig. 4 shows the effect of multiple reflection within the film. A, B, C, D are four successive images of the single point P on the source. The three loci obtained by taking these images successively in pairs intersect in a way very similar to those in the corresponding position in Fig. 3.

If the film be composed of a refracting medium, as water, instead of air, the light reflected from the lower surface of the film will appear to come, not from a point, but from a caustic surface; or, for a small pencil in any particular direction, from the circle of least confusion midway between two focal lines. Fig. 5 is an attempt to realize this condition graphically The caustic C represents the image of the point A on the source, formed by refraction at the upper surface of the film, and C' is its reflected image at the lower surface. For small angles of incidence, the effective image is quite near the cusp of the caustic, and the light which finally emerges from the film will them appear to come from points quite near the cusp of the caustic C''.

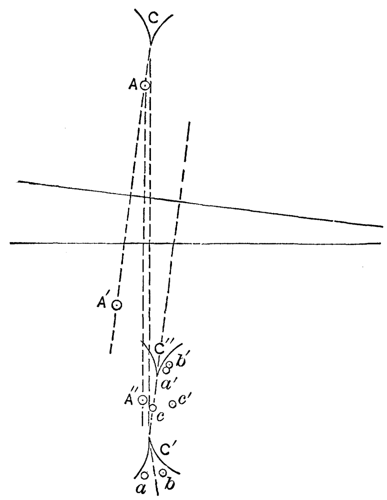

Fig. 5.

The substitution of water for air has thus moved the second image of A from A'' to the cusp of the caustic C'', which could also have been done by making the air film thinner, and of greater angle. Both of these changes would bring the fringes nearer to the film, for the same angle of incidence. The ratio of tangents of angles of film, multiplied by the ratio of thickness, deduced from the figure, is approximately one half. For small angles of incidence then the fringes should be about half as far from a water-film as from an air-film under similar conditions.

For a large angle of incidence, a and b may represent the effective images of A after reflection at the lower surface, for equal positive and negative values of the angle. For these two cases the cusp of the caustic C'' would be placed at c and c' respectively, and the final effective images of A, for light emerging from the upper surface, would be a' and b'. By the same reasoning that was used in the last paragraph, this shows that with increasing angle of incidence the distance of the water-fringes from the film does not increase so fast as that of the air-fringes. Also that the distance does not increase quite so fast for negative as for positive angles of incidence, negative angles being those for which the light is reflected away from the angle of the film. The drawing is hardly exact enough to show this last point with certainty, although the others are quite clear.

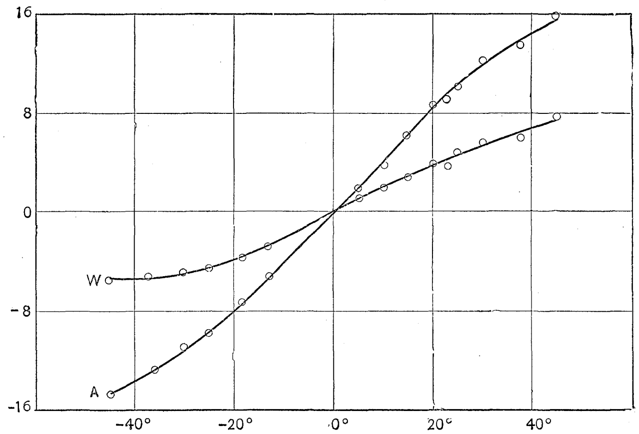

Fig. 6.

All of these conclusions were verified by a series of observations on a film enclosed between glass plates about 4 cm. long, in contact at one end and separated by two layers of tin-foil at the other. Small drops of water distributed irregularly in the film made it possible to observe the fringes in air and water, on either side of a boundary line, by merely changing the focus of the observing microscope without moving it laterally. The observations are plotted in Fig. 6, the curve *A* representing the results for the air-fringes and *W* for the water-fringes. Ordinates are distances in millimeters measured along the line of sight from film to fringes, and abscissæ are corresponding angles of incidence. It will be noted that for angles of incidence less than 20° the ratio of ordinates to the two curves is very nearly one half.

For angles of incidence greater than 45° the aberration becomes so great that measurements cannot be made with any accuracy.

It may be well to add that, for the sake of avoiding confusion, the phase difference of a half period introduced by the reflection within the glass, at the upper surface of the film, has been disregarded throughout.

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