A DETERMINATION OF THE VISCOSITY OF WATER.

By E. R. Drew.

THE apparatus used in this investigation is an attempt to realize as nearly as possible the ideal condition from which is obtained the conception of coefficient of viscosity, namely, that of a liquid contained between two parallel planes of infinite extent, one of which is moving in a direction parallel to the other, with constant velocity. It consists essentially of two vertical coaxial cylinders, the outer capable of rotating at any desired speed, while the inner, which is closed at the ends, is suspended by means of a torsion wire between agate bearings in the axis. The liquid used is contained in the space between the two cylinders.

The idea of coaxial cylinders was used for the same purpose by Professor Perry ("Liquid Friction," Phil. Mag., 35–441, 1893), who has given the complete mathematical theory, provided the effect due to the lower end of the suspended cylinder may be neglected or allowed for. The apparatus here used was constructed by Mr. E. L. Johonnott, under the direction of Professor Michelson; and as they were at the time unacquainted with the work of Perry, the idea was carried out in a quite different manner. Mr. Johonnott had completed the apparatus, and obtained some preliminary observations, when his attention was called to other lines of work, and the continuation of the investigation was intrusted to the writer by Professor Michelson.

The main object in view was to obtain a result which could be compared with those which have been obtained by the very different method, due to Poiseuille, of flow through capillary tubes.

Fig. 1 is a section through the axis of the instrument. The inner cylinder, A, is held in position by two agate bearings b, b, by means of which it can be centered with considerable accuracy. The torsion wire is attached to the cylinder by means of the rigid brass frame F, which passes around the heavy cross-bar carrying the upper bearing. When in use, the bearings and suspending wire are so adjusted as to give an amount of side play barely perceptible to the touch, in each bearing. The effects of friction in the bearings are then inappreciable. Deflections are read with telescope and scale, the latter in the form of an arc of a circle about the suspending wire as center.

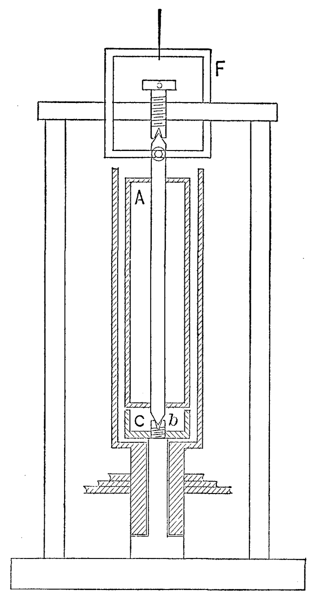

Fig. 1.

The inner cylinder was accurately turned; but as the inner surface of the outer cylinder could not readily be turned, with the facilities at hand, selected pieces of brass tubing which were very nearly cylindrical were used, and an attempt made to experimentally determine the error introduced. The outer cylinder is surrounded by a water-jacket, not shown in the figure, which allows the temperature of the liquid used to be readily determined, and aids in keeping it steady.

An ingenious and simple chronograph attachment, also not shown, was devised by Mr. Johonnott, to record the number of turns per second made by the outer cylinder during the time of an observation. Power was at first obtained from an engine in the mechanician's shop, but the speed was not sufficiently steady, and a 1-h.-p. electric motor was substituted, which, with a heavy balance-wheel on the countershaft, gave good satisfaction.

C is a brass cup screwed on the central shaft, with its upper edge almost in contact with the lower end of the suspended cylinder. It was found in practice that the water which filled the space between the cylinders seldom entered the cup. It was expected that this device would eliminate the effect, previously referred to, due to the lower end of the cylinder. When the cylinder had attained its equilibrium position, due to a steady rotation of the outer cylinder, the cup would form practically a continuation of the cylinder, of considerable length as compared with the thickness of the layer of liquid. Then the effect at a point near the lower end of the cylinder should be nearly or quite the same as at a point on an infinitely long cylinder, and Perry's analysis would apply directly.

A large number of observations showed that the deflection did not increase exactly with the speed, as it should, but somewhat faster. An example will show the magnitude of the effect. The speed is given under n, number of turns per second, and D is the deflection in cm. on the scale.

n	D	D/n
3.63	1.39	.383
5.20	2.01	.387
8.42	3.36	.399
10.61	4.37	.412
13.55	5.72	.422

To see whether this was in any way due to the free surface of the liquid, which was always set just at the upper edge of the suspended cylinder, a cup similar to C was suspended from the cross-bar, to form a continuation of the cylinder, and the free surface of the water raised some distance. The results then obtained showed no appreciable improvement. Subsequent experiments conducted along various lines seemed to indicate that the trouble was at the lower end of the cylinder, but no decisive result could be reached.

A little consideration showed that in order to apply the simple theory, as is done in Perry's discussion, the essential condition which must be fulfilled is that, when steady motion has been attained, all the points in the liquid which have a common speed must lie in a surface which, within the region considered, is everywhere equidistant from the bounding surfaces. Thus the surface of mean speed, in the case of a liquid contained between two planes, is a plane half way between them. In the present case, it is evident that the surface of mean speed will enter the angle P, Fig. 2, at an inclination of about 45° to the vertical, as shown by the dotted line, and will probably be displaced from its proper position for some distance above the lower end of the inner cylinder. If this displacement varies with the speed, the results obtained are accounted for.

The difficulty was overcome by means of the arrangement shown in Fig. 3. The two rings, shown in section at r and s, are attached, one to the outer cylinder, the other to the cup. The slant face of *s* is then fixed, while that of *r* moves with the outer cylinder, so that the vertical line midway between them represents the surface of mean speed. This is not exactly true, since different portions of the slant face of *r* move with different speeds ; but with the dimensions used,

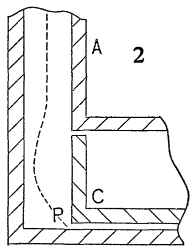

Fig. 2.

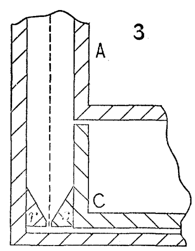

Fig. 3.

the error from this cause is small. With this arrangement the results at different speeds show no consistent difference.

The torsion constant of the suspending wire, defined as the force in dynes which must be applied at the surface of the cylinder, in a direction perpendicular to the axis, to produce a deflection of 1 cm. as read on the scale, was determined in two ways. The first method was that of the torsion pendulum, which took the form of a horizontal brass bar attached at its center to the lower end of the frame *F*. Near its ends, and equidistant from the center, two equal brass cylinders, with their axes vertical, were attached in such a way that they could be easily removed and replaced in the same position.

Following are the data for this determination :

Period of vibration, cylinders in position.	4.131 sec.
" " " " removed.	2.811 "
Moment of inertia of cylinders in position.	13780 gram-cm^2.
Radius of suspended cylinder *A*.	2.300 cm.
Scale distance.	76.52 cm.
Torsion constant, as previously defined.	161.3 dynes.

The plan of the second method is shown in Fig. 4; *acd* is a very fine silk fiber, one end of which is wound around the upper end of the suspended cylinder, the outer cylinder and the bearings having been removed. The other end is wound around a pin at *a*, and a small mass *m* is hung from *c* by a similar fiber. A third fiber is hung from *a* to serve as a plumb-line. The pin at *a* is attached to an arm pivoted near c, so that different inclinations of ac may be used. The portion cd of the fiber is brought to the horizontal each time by winding on the pin at a. The distances ab and bc are readon millimeter scales, and the horizontal force calculated. This force pulled the suspending wire out of the vertical, and slightly increased the scale-distance, from 76.52 cm. to 76.70 cm. in the extreme case, and this was allowed for in calculating results.

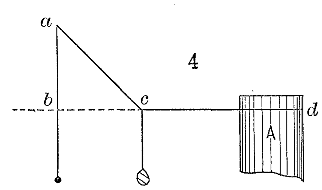

Fig. 4.

Torsion constant, mean of ten determinations, 161.4 dynes.

Greatest difference between two determinations, 0.6 dyne.

Two outer cylinders, of different diameters, were used. As they were not truly cylindrical, 16 equally-spaced diameters were measured at each end.

Dimensions of cylinders.	Outer Cylinder No. 1.	Outer Cylinder No. 2.	Inner Cylinder.
Mean radius, R_1.	2.538 cm.	2.851 cm.	R_2, 2.300 cm.
Greatest difference between two radii.	0.0023 cm.	0.015 cm.	
Length, L.			25.30 cm.

The final determinations were made at temperatures which seldom differed by more than 0°.2 from 20°, and the results were reduced to 20° by the formula computed by Helmholtz from Poiseuille's observations.[1]

$$\mu = \frac{0.0178}{1 + .0337t + .0022t^2}.$$

Observations were made at speeds of about 4, 6 and 8 turns per second. The result computed from each observation is the ratio D/n, the deflection which would be produced at a speed of one turn per second. These results are here summarized:

Outer Cylinder, No. 1.	
Mean of 33 observations.	$D/n = 1.615$
Greatest difference.	0.012
Average deviation from mean.	0.0031

Outer Cylinder, No. 2.	
Mean of 26 observations.	$D/n = 0.821$
Greatest difference.	0.009
Average deviation from mean.	0.0018
Scale distance.	76.18 cm.
Torsion constant of wire reduced to this distance.	160.6 dynes.

[1] See O. E. Meyer, Wied. Ann., 1877, p. 391.

The formula reached by Perry is

$$M = 4\pi\mu\omega R_1^2 R_2^2 / (R_1^2 - R_2^2)$$

where ω is the angular velocity of the outer cylinder, and M is the moment per unit length of the suspended cylinder, exerted upon its surface by a liquid for which μ is the coefficient of viscosity. Adapting this to the present case, it becomes

$$\mu = FDR_1(R_1^2 - R_2^2)/8\pi^2 L R_1^2 R_2^2$$

where D is the deflection at a speed of one turn per second, for which ω is 2π, and F is the torsion constant of the wire. The results are:

Outer cylinder, No. 1, $\mu = 0.01020$ dyne.
Outer cylinder, No. 2, $\mu = 0.01010$ dyne.

The lower end of the outer cylinder slipped somewhat loosely into a sleeve which kept it centered at the bottom, while allowing it to be tipped about slightly, and centered at the top by adjustment. In each case a set of observations was made with the top out of center by a measured amount, in order to obtain an approximate value for the error arising from the fact that the outer cylinder was not truly cylindrical. When the two cylinders are set coaxially, the greatest difference between two radii gives δt, the greatest difference in thickness of the liquid layer. At a given point on the surface of the inner cylinder this change in thickness would occur twice in each revolution. When the outer cylinder is tipped so that its upper end is off center, the greatest change in thickness, $\delta_1 t$, occurs but once in each revolution. Assuming that for such small variations, the error is proportional to the first power of the effective δt, the error in the first case would be proportional to $2\delta t$, and in the second to $2\delta t + \delta_1 t/2$. In the table of results, D_1 is the deflection obtained when the outer cylinder was tipped.

	D	D_1	δt	$\delta_1 t$
Outer cylinder No. 1.	1.615	1.595	0.0023 cm.	0.03 cm.
" " " 2.	0.821	0.813	0.0150 cm.	0.05 cm.

This gives the following corrections to be applied to the results:

	Correction.	Corrected μ.
No. 1.	+ 0.3 per cent.	0.01023
" 2.	+ 1.0 " "	0.01020

The thermometer used had not been calibrated, and Professor Ames, of Johns Hopkins University, very kindly arranged to have it compared with a Tonnelot standard thermometer. The comparison, made by Mr. Chas. Waidner, showed that at 20°, the temperature of the above determinations, the thermometer differed from the Paris nitrogen standard by 0°.03, somewhat less than the probable error of a reading. So that no temperature correction is necessary.

To compare with these results, the value of the coefficient from the observations of Poiseuille, computed by the formula previously given, is $\mu = 0.0101$.

RYERSON PHYSICAL LABORATORY, UNIVERSITY OF CHICAGO, March, 1898.