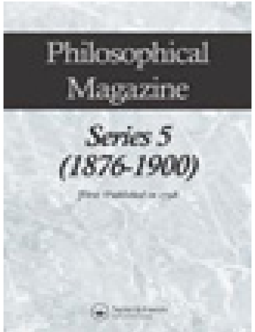

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I. On the application of interference methods to astronomical measurements

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[FIFTH SERIES.]

JULY 1890.

I. *On the Application of Interference Methods to Astronomical Measurements.* By ALBERT A. MICHELSON*.

[Plates I. & II.]

IN a recent paper on "Measurement by Light-Waves" † it was shown that the limitation of the effective portions of an objective to the extreme ends of a diameter converted the instrument into a refractometer; and although definition and resolution are thereby sacrificed, the accuracy may be increased ten to fifty fold.

The simplest way of effecting this in the case of a telescope is to provide the cap of the objective with two slits adjustable in width and distance apart. If such a combination be focused on a star, then, instead of an image of the star, there will be a series of coloured interference-bands with white centre, the bands being arranged at equal distances apart and parallel to the two slits. The position of the central white fringe can be marked from ten to fifty times as accurately as can the centre of the telescopic image of the star.

One of the most promising applications of the method is the measurement of the angular magnitudes of small sources of light.

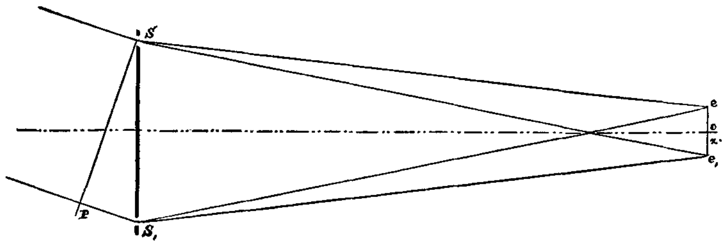
This may be accomplished by taking advantage of the well-known principle that in order to obtain clear interference-

* Communicated by the Author.

† 'American Journal of Science,' xxxix. Feb. 1890.

bands from two pencils diverging from the same source (width a) at an angle β, it is necessary that either β or a be very small.

Fig. 1.



Thus, in fig. 1, let us take

$a =$ ee_1 $=$width of the source.
$d =$ distance of source from the objective.
$b =$ SS_1=distance between the slits.

Also put $S_1P = \Delta$, $\frac{a}{d} = \alpha$, $\frac{b}{d} = \beta$.

Then the usual statement is that the interference-fringes vanish when $Se_1 - Se = \frac{1}{2}\beta a = \frac{1}{2}b\alpha = \frac{1}{2}\lambda$, or when

$$\alpha = \frac{\lambda}{b}.$$

But $\frac{\lambda}{b}$ is the "limit of resolution" of the telescope of aperture b, and if this be denoted by α_0 we have

$$\alpha = \alpha_0.$$

Or, in words, the fringes disappear when the source subtends an angle which can just be resolved by the telescope.

The experiment was first tried with an objective of 45 millim. effective diameter (distance between the slits) at a distance of ten metres from an adjustable slit which served as the source.

It was found that the first indication of indistinctness occurred when a was 0·08 millim. wide, and at 0·14 millim. the fringes almost vanished.

But on continuing to widen the slit they again became clearly visible, to disappear and reappear at regular intervals.

Now, though it might with truth be urged that the observation of the indefinite vanishing of interference-fringes depends so much on the attendant circumstances, and especially on the condition of the observer, that it can scarcely be called a precise measurement, yet the statement applies no longer when the disappearance depends on the existence of well-marked *minima of distinctness*; and, as will appear below, it is possible to measure, with accuracy, by the observation of these minima the width of a source of light, which in a telescope can with difficulty be ascertained to have an appreciable size.

The theory of these successive appearances and disappearances is as follows:—

Returning to fig. 1, let x be the distance of any element of the source from the axis of the telescope, dx the width of the element, and $y=\phi(x)$ the length.

Then the difference in the two paths xS and $x\mathrm{S}_1\mathrm{P}$ terminating at the wave-front P, which makes an angle γ with the plane perpendicular to the axis of the telescope, will be $\beta x-\gamma b$, and the resulting intensity in the direction γ for the whole source will therefore be

$$\mathrm{I}=\int\phi(x)\left[1+\cos\frac{2\pi}{\lambda}(\beta x-\gamma b)\right]dx. \quad . \quad . \quad . \quad (1)$$

Case I.—*Uniformly Illuminated Slit.*

If the source be a slit whose centre is in the axis, and whose length is parallel to the slits SS_1, and whose width is a, then

$$\mathrm{I}=a+\frac{\lambda}{\pi\beta}\sin\frac{\pi\beta}{\lambda}a\cos\frac{2\pi\gamma}{\lambda}b. \quad . \quad . \quad . \quad . \quad (2)$$

If I_1 be the intensity at the centre of a bright fringe, and I_2 that at the centre of a dark fringe, then the visibility of the fringes may be expressed by

$$\mathrm{V}=\frac{\mathrm{I}_1-\mathrm{I}_2}{(\mathrm{I}_1+\mathrm{I}_2)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But

$$\mathrm{I}_1=a+\frac{\lambda}{\pi\beta}\sin\frac{\pi\beta}{\lambda}a,$$

$$\mathrm{I}_2=a-\frac{\lambda}{\pi\beta}\sin\frac{\pi\beta}{\lambda}a;$$

$$\therefore \qquad \mathrm{V}=\frac{\sin\frac{\pi\beta}{\lambda}a}{\frac{\pi\beta}{\lambda}a}$$

or, finally,

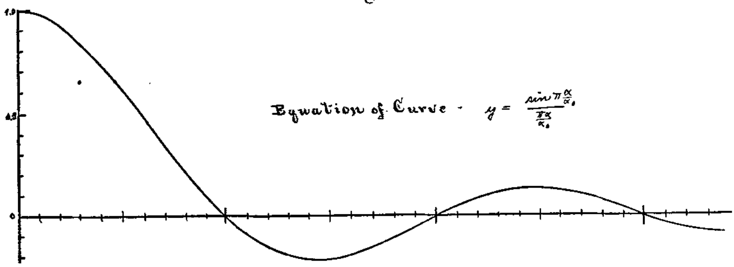
$$V = \frac{\sin \pi \frac{\alpha}{\alpha_0}}{\pi \frac{\alpha}{\alpha_0}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence the fringes will disappear whenever α is a multiple of α_0. They will be clearest when

$$\alpha = \frac{\alpha_0}{\pi} \tan \pi \frac{\alpha}{\alpha_0} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The curve of visibility will be of the form given in fig. 2.

Fig. 2.



The successive values of the maxima, disregarding signs, calculated from the above equation, are :—

$$V = +\ 1{\cdot}000$$
$$V = -\ 0{\cdot}210$$
$$V = +\ 0{\cdot}130$$
$$V = -\ 0{\cdot}091$$

The negative values mean that during the corresponding period the fringes are *reversed* *.

As the expression for V contains λ, the wave-length of the light employed, the fringes can disappear for that colour (and the complementary one) only; and as this will be most decided for the brightest colour in the spectrum, the yellow or greenish yellow (and therefore also for violet), there will be a residue of red and blue-green mixed with much white light. Both of these deductions have been fully verified by experiment.

* Probably the expression for V should be $\left(\frac{I_1 - I_2}{I_1 + I_2}\right)^2$, but this will not affect the position of the points for which $V = 0$.

Table I. shows well the accordance between theory and observation. Under S is given the nature of the light used: S denotes sunlight, C calcium light, R red light.

The other quantities given in the Table are :—

λ = Approximate wave-length in thousandths of a millimetre.
a = Width of slit by direct measurement.
d = Distance to telescope.
b = Distance between centres of slits.
f = Factor depending on the order of the observed disappearance.
α = Angle deduced from the observations.

$$\alpha_1 = \frac{a}{d} \times 206265''.$$

e = Error in seconds.
o = Error per cent.

TABLE I.

S.	λ.	*a.*	*d.*	*b.*	*f.*	*α.*	*α*₁.	*e.*	*o.*
S.	·550	·240	20,800	50·15	1·00	2″·26	2″·38	−0″·12	− 5
,,	,,	·490	,,	23·35	1·00	4·86	4·85	+0·01	0
,,	,,	·740	,,	15·45	1·00	7·33	7·33	·00	0
C.	·570	·197	20,340	65·75	1·00	1·78	1·99	− ·21	−10
,,	,,	·365	,,	65·75	2·00	3·57	3·69	− ·12	− 3
,,	,,	·547	,,	65·75	3·00	5·35	5·53	− ·18	− 3
,,	,,	·723	,,	65·75	4·00	7·14	7·31	− ·17	− 2
,,	,,	·900	,,	65·75	5·00	8·92	9·11	− ·19	− 2
,,	,,	·238	,,	53·75	1·00	2·18	2·41	− ·23	−10
,,	,,	·442	,,	53·75	2·00	4·36	4·47	− ·11	− 3
,,	,,	·650	,,	53·75	3·00	6·55	6·58	− ·03	0
,,	,,	·865	,,	53·75	4·00	8·73	8·75	− ·02	0
,,	,,	1·105	,,	53·75	5·00	10·91	11·18	− ·27	− 3
,,	,,	·277	,,	40·75	1·00	2·88	2·80	+ ·08	+ 3
,,	,,	·567	,,	40·75	2·00	5·76	5·74	+ ·02	0
,,	,,	·858	,,	40·75	3·00	8·64	8·68	− ·04	0
,,	,,	1·150	,,	40·75	4·00	11·52	11·64	− ·12	− 1
,,	,,	1·412	,,	40·75	5·00	14·40	14·28	+ ·12	+ 1
,,	,,	·388	,,	31·15	1·00	3·77	3·93	− ·16	− 4
,,	,,	·757	,,	31·15	2·00	7·54	7·66	− ·12	− 2
,,	,,	1·160	,,	31·15	3·00	11·31	11·73	− ·42	− 4
,,	,,	1·532	,,	31·15	4·00	15·08	15·40	− ·32	− 2
,,	,,	1·902	,,	31·15	5·00	18·85	19·25	− ·40	− 2
R.	·623	·298	,,	44·75	1·00	2·86	3·02	− ·16	− 5
,,	,,	·573	,,	44·75	2·00	5·73	5·80	− ·07	− 1
,,	,,	·852	,,	44·75	3·00	8·65	8·62	+ ·03	0
,,	,,	1·108	,,	44·75	4·00	11·46	11·21	+ ·25	+ 2
,,	,,	1·415	,,	44·75	5·00	14·32	14·32	·00	0
,,	,,	1·672	,,	44·75	6·00	17·18	16·92	+ ·26	+ 2
,,	,,	1·945	,,	44·75	7·00	20·05	19·69	+ ·36	+ 2
,,	,,	2·223	,,	44·75	8·00	22·91	22·49	+ ·42	+ 2

It may be noted that under the observations marked R no less than eight successive disk disappearances were noted, the average error being less than 2 per cent.

CASE II.—*Uniformly Illuminated Disk.*

If the source be a uniformly illuminated disk of radius r, the expression to be integrated is

$$I=4\int_0^r \sqrt{r^2-x^2}\left[1+\cos\frac{2\pi}{\lambda}(\beta x-\gamma b)\right]dx. \quad . \quad (6)$$

Putting α = angular diameter.
Then

$$\frac{\beta}{\lambda}=\frac{\alpha}{\alpha_0}\,\frac{1}{r}.$$

Write

$$\frac{x}{r}=w, \text{ and } \pi\frac{\alpha}{\alpha_0}=n.$$

Then omitting the phase constant γb this reduces to the form

$$I=\frac{\pi}{4}\left[1\pm\frac{4}{\pi}\int_0^1\sqrt{1-w^2}\cos nw\,dw\right], \quad . \quad . \quad (7)$$

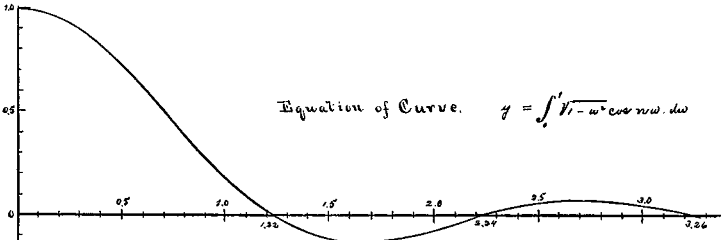
in which the integral is positive for the maxima and negative for the minima.

Putting

$$\int_0^1\sqrt{1-w^2}\cos nw\,dw=A, \text{ we have for V,}$$

$$V=A. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Fig. 3.



Tables for A are given in Airy's "Undulatory Theory," and give the results expressed in the curve shown in fig. 3.

The values of $\frac{\alpha}{\alpha_0}$ for which the fringes disappear are

1·22, 2·24, 3·26, 4·26, &c.,

and the values of the successive (positive or negative) maxima are :—

$$V = +1{\cdot}000$$
$$V = -0{\cdot}130$$
$$V = +0{\cdot}065$$
$$V = -0{\cdot}040$$

Here, too, the theory is remarkably well confirmed by experiment. This is shown by Table II., in which the letters have the same meaning as before, save that *a* is now the diameter of the circular aperture.

TABLE II.

S.	λ.	*a.*	*d.*	*b.*	*f.*	*α.*	α_1.	*e.*	*o.*
S.	·550	·701	20,800	20·15	1·22	6″·85	6″·94	−0″·09	−1
C.	·570	·200	20,340	77·71	1·22	1·85	2·02	− ·17	−8
,,	,,	·271	,,	51·95	1·22	2·76	2·73	+ ·03	+1
,,	,,	·445	,,	32·50	1·22	4·41	4·50	− ·09	−2
,,	,,	·445	,,	52·45	2·24	4·52	4·50	+ ·02	0
,,	,,	·445	,,	81·45	3·26	4·60	4·50	+ ·10	+2
,,	,,	·701	,,	20·00	1·22	7·17	7·09	+ ·08	+1
,,	,,	·701	,,	34·85	2·24	7·55	7·09	+ ·46	+6
,,	,,	·701	,,	52·99	3·26	7·14	7·09	+ ·05	+1
,,	,,	1·000	,,	25·65	2·24	10·20	10·12	+ ·08	+1
,,	,,	1·000	,,	14·65	1·22	9·77	10·12	− ·35	−4
,,	,,	1·000	,,	27·35	2·24	9·63	10·12	− ·49	−5
,,	,,	1·000	,,	38·55	3·26	9·87	10·12	− ·25	−2
,,	,,	1·230	,,	21·08	2·24	12·47	12·44	+ ·03	0
,,	,,	1·230	,,	30·45	3·26	12·58	12·44	+ ·14	+1
,,	,,	1·230	,,	11·15	1·22	12·85	12·44	+ ·41	+3
,,	,,	1·230	,,	21·45	2·24	12·23	12·44	− ·21	−2
,,	,,	1·230	,,	31·15	3·26	12·25	12·44	− ·19	−2
,,	,,	1·230	,,	39·95	4·26	12·49	12·44	+ ·05	0
,,	,,	1·465	,,	17·42	2·24	14·92	14·82	+ ·10	+1
,,	,,	1·465	,,	25·99	3·26	14·70	14·82	− ·12	−1
,,	,,	1·690	,,	15·14	2·24	17·36	17·10	+ ·26	+2
,,	,,	1·690	,,	23·40	3·26	16·36	17·10	− ·74	−4
R.	·623	0·445	,,	35·15	1·22	4·45	4·50	− ·05	−1

The curve V = A (fig. 3) shows that the maxima are considerably smaller than in the case of the slit, and, accordingly, the disappearances are not quite so sharp ; but in one case it was possible to note four of them, and in this case also the average error was less than 2 per cent.

CASE III.—*Illumination not Uniform.*

On applying the preceding formulæ to observations on images of the sun's disk the angles obtained were all too small by about 10 per cent., the cause of the discrepancy being the want of uniformity of illumination.

The curve (fig. 4) represents the intensity at any point of the sun's disk as a function of its distance from the centre,

$$i = f\,(x^2 + y^2)^{\frac{1}{2}}.$$

The ordinates are taken from Vogel's Table*.

Fig. 4.

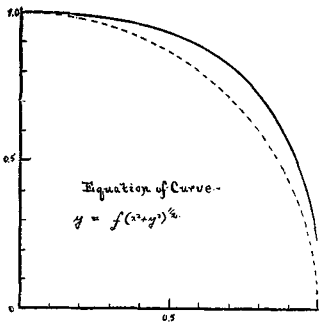
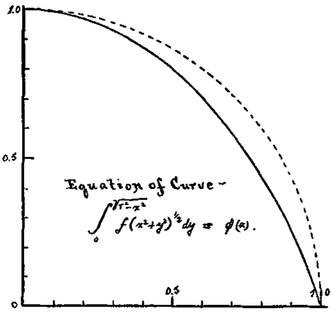


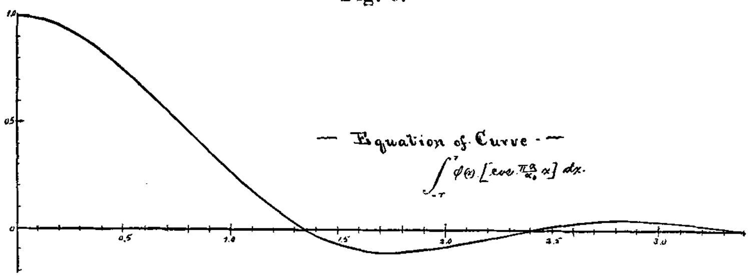
Fig. 5.



From this curve we obtain that shown in fig. 5 by integrating the expression

$$\int_0^{\sqrt{r^2-x^2}} f(\sqrt{x^2+y^2})\,dy = \phi(x). \quad . \quad . \quad . \quad (9)$$

Fig. 6.



This value of $\phi(x)$ substituted in (1) gives on integration

* Young on 'The Sun.'

the value of the intensity of the maxima and minima in the form $I=1\pm B$, whence finally we get for the visibility of the fringes

$$V = B. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

The values of V found from equation (10) are plotted in the curve shown in fig. 6 *.

The fringes disappear at the points for which $\frac{\alpha}{\alpha_0}$ is

$$1{\cdot}33, \ 2{\cdot}38, \ \&c.;$$

and the values of the successive (positive and negative) maxima are:—

$$V = +1{\cdot}00$$
$$V = -0{\cdot}1$$
$$V = +0{\cdot}042$$
$$V = -0{\cdot}028$$

On applying these results to the observations the accordance is very satisfactory, as appears from the following Table :—

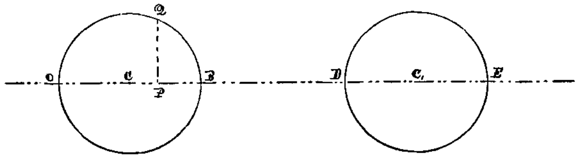
TABLE III.

S.	λ.	*a.*	*d.*	*b.*	*f.*	*α.*	α_1.	*e.*	*o.*
S.	·550	·231	27,800	84·75	1·33	1″·74	1″·71	+0″·03	+2
,,	,,	·377	,,	55·25	1·33	2·72	2·79	− ·07	−2
,,	,,	·933	,,	22·65	1·33	6·66	6·90	− ·24	−3
,,	,,	·231	21,200	66·15	1·33	2·27	2·24	+ ·03	+1
,,	,,	·377	,,	41·75	1·33	3·59	3·66	− ·07	−2
,,	,,	·933	,,	16·65	1·33	9·04	9·05	− ·01	0

CASE IV.—*Double Source.*

Suppose next that there are two equal symmetrical sources of light whose centres are at C and C_1 (fig. 7).

Fig. 7.



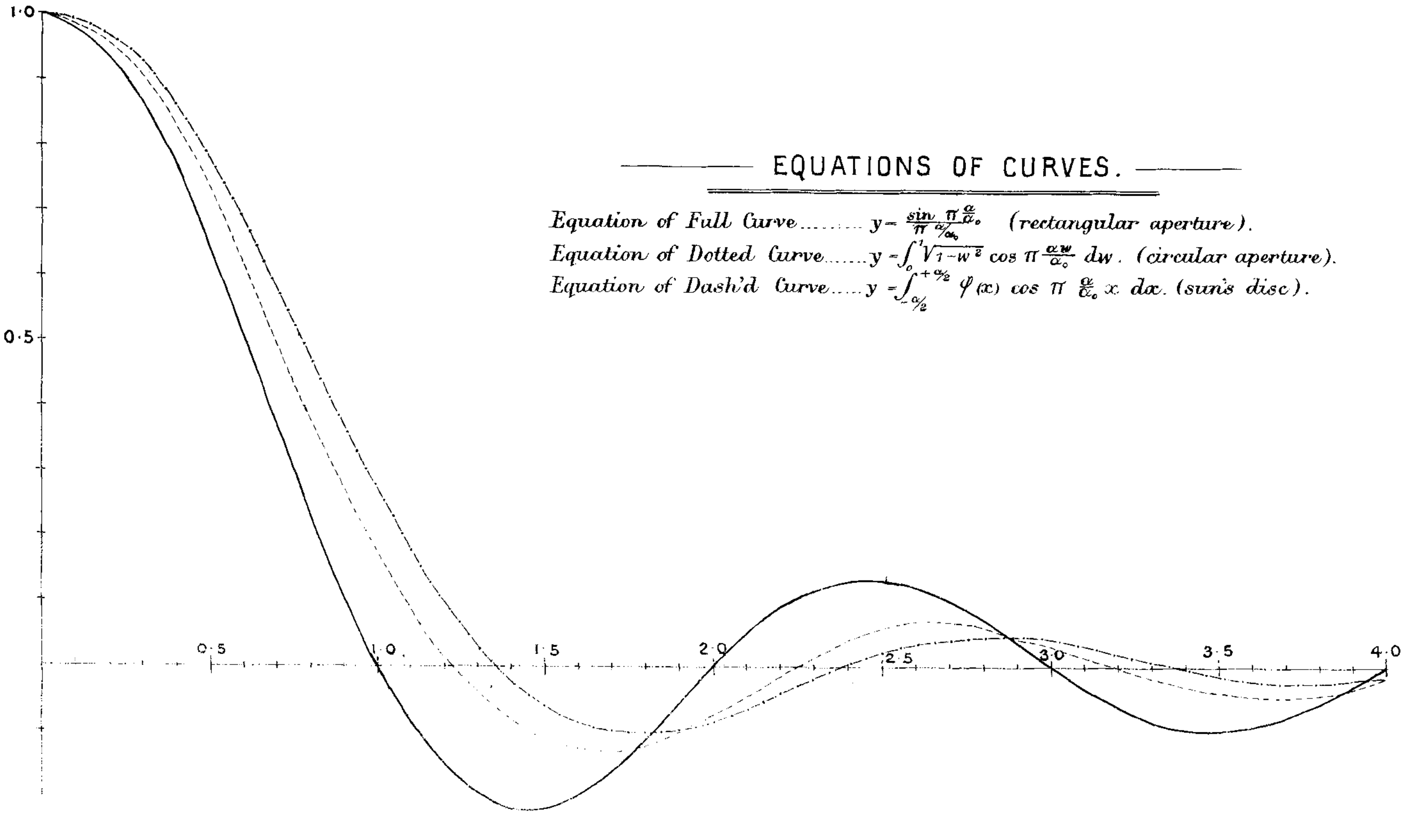
* The curves corresponding to Cases I., II., and III. are for convenience of comparison given together in Plate I.

EQUATIONS OF CURVES.

Equation of Full Curve......... $y = \frac{\sin \pi \frac{\alpha}{\alpha_0}}{\pi \, \alpha/\alpha_0}$ (rectangular aperture).

Equation of Dotted Curve.......$y = \int_0^1 \sqrt{1-w^2} \cos \pi \frac{\alpha w}{\alpha_0} \, dw$. (circular aperture).

Equation of Dash'd Curve.....$y = \int_{-\alpha/2}^{+\alpha/2} \varphi(x) \cos \pi \frac{\alpha}{\alpha_0} x \, dx$. (sun's disc).



Mintern Bros. lith.

Let $OC = CB = DC_1 = C_1E = r$.

$CC_1 = 2s, \quad OP = x, \quad PQ = y.$

Then the intensity of the interference-fringes will be

$$I = \int y\left(1 + \cos\frac{2\pi\Delta}{\lambda}\right)dx \ldots \text{ as in (1).}$$

Integrating from O to B and from D to E, we have

$$I = \int_0^{2r} y_1\left(1 + \cos\frac{2\pi\Delta}{\lambda}\right)dx + \int_{2s}^{2s+2r} y_2\left(1 + \cos\frac{2\pi\Delta}{\lambda}\right)dx; \quad (11)$$

where

$$y_1 = f(x-r) \text{ and } y_2 = f[x-(2s+r)]$$

and

$$\Delta = \beta x - \gamma b.$$

In the first integral put $w_1 = (x-r)$.

" second " $w_2 = x-(2s+r)$.

Then we obtain

$$I = \int_{-r}^{+r} f(w_1)\left[1 + \cos\frac{2\pi}{\lambda}(\beta w_1 + \beta r - \gamma b)\right]dw_1$$

$$+ \int_{-r}^{+r} f(w_2)\left[1 + \cos\frac{2\pi}{\lambda}(\beta w_2 + \beta r - \gamma b + 2\beta s)\right]dw_2. \quad . \quad (12)$$

Expanding the first of these integrals we obtain :—

$$\int_{-r}^{+r} f(w_1)\,dw_1 + \cos\frac{2\pi}{\lambda}(\beta r - \gamma b)\int_{-r}^{+r} f(w_1)\cos\frac{2\pi}{\lambda}\beta w_1\,dw_1$$

$$-\sin\frac{2\pi}{\lambda}(\beta r - \gamma b)\int_{-r}^{+r} f(w_1)\sin\frac{2\pi}{\lambda}(\beta w_1)\,dw_1;$$

in which the first term is half the area of the aperture, and the last term (since $f(w_1)$ is a symmetrical function) is 0. The same is also true of the expansion of the second integral. If, then, we put

$$\int_{-r}^{+r} f(w)\,dw = \tfrac{1}{2}Q,$$

$$\int_{-r}^{+r} f(w)\cos\frac{2\pi}{\lambda}\beta w\,dw = \tfrac{1}{2}QA,$$

equation (12) becomes

$$I=Q+\tfrac{1}{2}QA\cos\frac{2\pi}{\lambda}(\beta r-\gamma b)+\tfrac{1}{2}QA\cos\frac{2\pi}{\lambda}(\beta r-\gamma b+2\beta s)$$

or

$$I=Q+QA\left(\cos\frac{2\pi}{\lambda}[\beta s+\beta r-\gamma b]\cos\frac{2\pi}{\lambda}\beta s\right); \quad . \quad . \quad . \quad (13)$$

whence for the visibility

$$V=\frac{I_1-I_2}{I_1+I_2}=A\cos\frac{2\pi}{\lambda}\beta s,$$

or finally

$$V=A\cos\pi\frac{\alpha}{\alpha_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

1st. When the sources are two equal uniformly illuminated slits of height $2h$,

$$f(w)=h, \text{ and } Q=4rh.$$

Hence

$$A=\frac{1}{r}\int_0^r\cos\frac{2\pi\beta}{\lambda}w\,dw=\frac{\sin\frac{2\pi}{\lambda}\beta r}{\frac{2\pi}{\lambda}\beta r} \quad . \quad . \quad . \quad . \quad (15)$$

Putting $\frac{2\pi}{\lambda}\beta r=\pi\frac{\alpha_1}{\alpha_0}$, and substituting for A in equation (14),

$$V=\frac{\sin\pi\frac{\alpha_1}{\alpha_0}}{\pi\frac{\alpha_1}{\alpha_0}}\cos\pi\frac{\alpha}{\alpha_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

2nd. For the case of two equal uniformly illuminated circular apertures of radius r,

$$f(w)=\sqrt{r^2-w^2}, \text{ and } Q=\pi r^2.$$

Hence

$$A=\frac{4}{\pi r^2}\int_0^r\sqrt{r^2-w^2}\cos\frac{2\pi}{\lambda}\beta w\,dw.$$

Putting $\frac{w}{r}=z$, this reduces to the form already given for Airy's integral, and the expression for the visibility of the fringes is

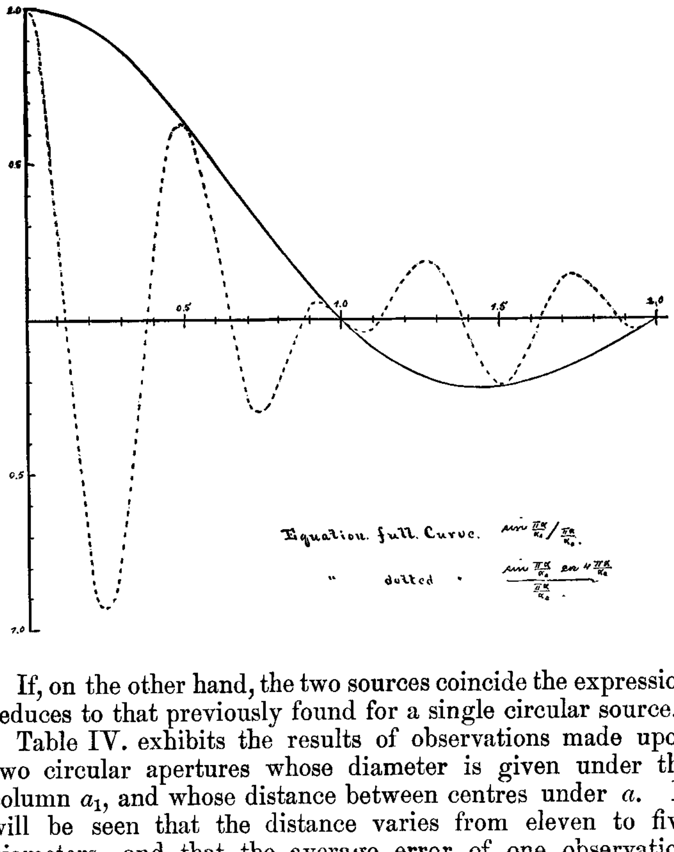
$$V=A_1\cos\pi\frac{\alpha}{\alpha_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

When the distance between the sources is more than five or six times the width, the periodicity of the second term alone is of importance, the term A_1 representing the amplitude of its variation.

The fringes vanish whenever $\alpha = \frac{2n-1}{2}\alpha_0$.

The general form of the visibility curve is given in fig. 8.

Fig. 8.



If, on the other hand, the two sources coincide the expression reduces to that previously found for a single circular source.

Table IV. exhibits the results of observations made upon two circular apertures whose diameter is given under the column a_1, and whose distance between centres under a. It will be seen that the distance varies from eleven to five diameters, and that the average error of one observation (usually a single setting) is only $1\frac{1}{2}$ per cent.

TABLE IV.

S.	λ.	a_1.	a.	b.	f.	α.	α_1.	e.	o.
C.	·560	·060	·693	25·1	1·50	6″·90	6″·80	+0″·10	+2
,,	,,	·060	·693	25·3	1·50	6·85	6·80	+ ·05	+1
,,	,,	·060	·693	25·7	1·50	6·75	6·80	− ·05	−1
,,	,,	·060	·693	25·7	1·50	6·75	6·80	− ·05	−1
,,	,,	·060	·666	26·6	1·50	6·53	6·58	− ·05	−1
,,	,,	·060	·666	26·7	1·50	6·50	6·58	− ·08	−1
,,	,,	·060	·519	34·4	1·50	5·00	4·99	+ ·01	0
,,	,,	·060	·465	12·9	0·50	4·48	4·56	− ·08	−2
,,	,,	·060	·315	18·8	0·50	3·07	3·09	− ·02	−1
R.	·623	·060	·693	27·8	1·50	6·93	6·80	+ ·13	+2
,,	,,	·060	·693	27·9	1·50	6·90	6·80	+ ·10	+2
,,	,,	·060	·666	29·5	1·50	6·53	6·58	− ·05	−1
,,	,,	·060	·666	29·4	1·50	6·57	6·58	− ·01	0
S.	·550	·117	·526	10·75	0·50	5·27	5·18	+ ·09	+2
,,	,,	·117	·526	11·65	0·50	4·86	5·03	− ·17	−3
C.	·560	·117	·526	10·85	0·50	5·32	5·32	·00	0
,,	,,	·117	·526	33·24	1·50	5·21	5·32	− ·11	−2
,,	,,	·117	·526	55·80	2·50	5·18	5·32	− ·14	−3
,,	,,	·117	·526	80·15	3·50	5·08	5·32	− ·24	−4
R.	·623	·117	·526	36·02	1·50	5·34	5·32	+ ·02	0
,,	,,	·117	·526	60·20	2·50	5·32	5·32	·00	0
,,	,,	·117	·526	85·70	3·50	5·24	5·32	− ·08	−2

When the distance between centres is two diameters or less, it is found that the results obtained are much less accurate, being usually too large by about 4 per cent., as shown in Table V.

TABLE V.

S.	.	a_1.	a.	b.	f.	α.	α_1.	e.	o.
C.	·560	·184	·396	42·8	1·50	3″·99	3″·89	+0″·10	+3
,,	,,	·184	·372	14·8	0·50	3·81	3·66	+ ·15	+4
,,	,,	·184	·363	16·9	0·50	3·40	3·54	− ·14	−4
,,	,,	·184	·332	16·6	0·50	3·41	3·22	+ ·19	+6
,,	,,	·184	·297	19·6	0·50	2·90	2·88	+ ·02	+1
,,	,,	·184	·266	21·3	0·50	2·71	2·60	+ ·11	+4
,,	,,	·184	·266	20·4	0·50	2·83	2·62	+ ·21	+8
,,	,,	·184	·251	23·3	0·50	2·48	2·47	+ ·01	0
,,	,,	·184	·251	22·8	0·50	2·51	2·47	+ ·04	+2
,,	,,	·184	·227	24·9	0·50	2·32	2·23	+ ·09	+4
,,	,,	·184	·219	25·6	0·50	2·22	2·13	+ ·09	+4
S.	·550	·060	·107	56·2	0·50	1·01	1·06	− ·05	−5
,,	,,	·060	·107	56·8	0·50	1·02	1·04	− ·02	−2
C.	·560	·060	·107	54·0	0·50	1·07	1·08	− ·01	−1
,,	,,	·060	·107	53·2	0·50	1·09	1·05	+ ·04	+4
,,	,,	·060	·107	53·5	0·50	1·08	1·05	+ ·03	+3
R.	·623	·060	·107	57·2	0·50	1·13	1·05	+ ·08	+8
,,	,,	·060	·107	58·7	0·50	1·10	1·05	+ ·05	+5
C.	·560	·060	·075	70·8	0·50	0·79	0·74	+ ·05	+7
,,	,,	·060	·075	72·0	0·50	0·77	0·74	+ ·03	+4

The measurement of the angular magnitude of sources of light too small to be resolved by the telescope may also be effected with a considerable degree of accuracy by providing the telescope objective with a slit or diaphragm which can be varied in width or size, and measuring the distance between the centres of the diffraction-fringes by means of a micrometer eyepiece. In case the slit is used (as found most convenient in practice), the expression for the intensity of illumination at a distance γ from the centre of the central bright band will be

$$I=\int f(\phi)\frac{\sin^2\frac{\pi}{\alpha_0}(\gamma-\phi)}{\left[\frac{\pi}{\alpha_0}(\gamma-\phi)\right]^2}d\phi, \quad . \quad . \quad . \quad (18)$$

where $f(\phi)=y$ is the ordinate of source of light (of uniform intensity) at the distance ϕ from the axis. If the source has a width α, the limits of the integral are $\frac{\alpha}{2}$ and $-\frac{\alpha}{2}$.

1st. In case the source is a uniformly illuminated slit, $f(\phi)=\text{const.}$, and the expression for I becomes

$$I=C\int_{-\alpha/2}^{+\alpha/2}\frac{\sin^2\frac{\pi}{\alpha_0}(\gamma-\phi)}{\left[\frac{\pi}{\alpha_0}(\gamma-\phi)\right]^2}d\phi,$$

or, substituting $\frac{\pi}{\alpha_0}(\gamma-\phi)=x$,

$$I=C_1\int_{\pi/\alpha_0(\gamma-\alpha/2)}^{\pi/\alpha_0(\gamma+\alpha/2)}\frac{\sin^2 x}{x^2}dx. \quad . \quad . \quad . \quad . \quad (19)$$

This integral cannot be found directly, but we may obtain the values of γ for which I is a maximum or a minimum by differentiating (19) with respect to γ.

This gives

$$\frac{dI}{d\gamma}=\text{Const.}\left[\frac{\sin^2\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}{\left[\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)\right]^2}-\frac{\sin^2\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}{\left[\frac{\pi}{\alpha_0}\left(\gamma-\frac{\alpha}{2}\right)\right]^2}\right]=0.$$

Hence
$$\frac{\sin\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}{\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}=\pm\frac{\sin\frac{\pi}{\alpha_0}\left(\gamma-\frac{\alpha}{2}\right)}{\frac{\pi}{\alpha_0}\left(\gamma-\frac{\alpha}{2}\right)}. \quad . \quad . \quad (20)$$

Writing this in the form

$$\frac{\sin\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}{\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}=\pm\frac{\sin\left[\left(\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)-\pi\frac{\alpha}{\alpha_0}\right)\right]}{\frac{\pi}{\alpha_0}\left(\gamma-\frac{\alpha}{2}\right)},$$

we see that when $\frac{\alpha}{\alpha_0}<1$ the two sides of the equation have opposite signs, because

$$\frac{\pi}{\alpha_0}\left(\gamma-\frac{\alpha}{2}\right)<\pi<\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right);$$

and we have

$$\frac{\sin\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}{\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}=-\frac{\sin\frac{\pi}{\alpha_0}\left(\gamma-\frac{\alpha}{2}\right)}{\frac{\pi}{\alpha_0}\left(\gamma-\frac{\alpha}{2}\right)}. \quad . \quad . \quad (21)$$

Expanding and solving for γ we find

$$\gamma\tan\frac{\pi\gamma}{\alpha_0}=\frac{\alpha}{2}\tan\frac{\pi\alpha}{2\alpha_0}, \quad . \quad . \quad . \quad . \quad (22)$$

which gives the condition for a minimum when

$$0<\frac{\alpha}{\alpha_0}<1,\quad 2<\frac{\alpha}{\alpha_0}<3,\ldots 2m<\frac{\alpha}{\alpha_0}<2m+1\ldots$$

When $1<\frac{\alpha}{\alpha_0}<2$, the sign of the two sides is the same, and we have

$$\frac{\sin\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}{\frac{\pi}{\alpha_0}\left(\gamma+\frac{\alpha}{2}\right)}=\frac{\sin\frac{\pi}{\alpha_0}\left(\gamma-\frac{\alpha}{2}\right)}{\frac{\pi}{\alpha_0}\left(\gamma-\frac{\alpha}{2}\right)};$$

and solving for γ as before we find

$$\frac{\tan\frac{\pi\gamma}{\alpha_0}}{\gamma}=\frac{\tan\frac{\pi\alpha}{2\alpha_0}}{\frac{\alpha}{2}}, \quad . \quad . \quad . \quad . \quad . \quad (23)$$

which gives the conditions for a minimum for

$$1<\frac{\alpha}{\alpha_0}<2,\quad 3<\frac{\alpha}{\alpha_0}<4\ldots 2m-1<\frac{\alpha}{\alpha_0}<2m.$$

Finally, when $\dfrac{\alpha}{\alpha_0}=1, 3, 5, \ldots 2m-1,$

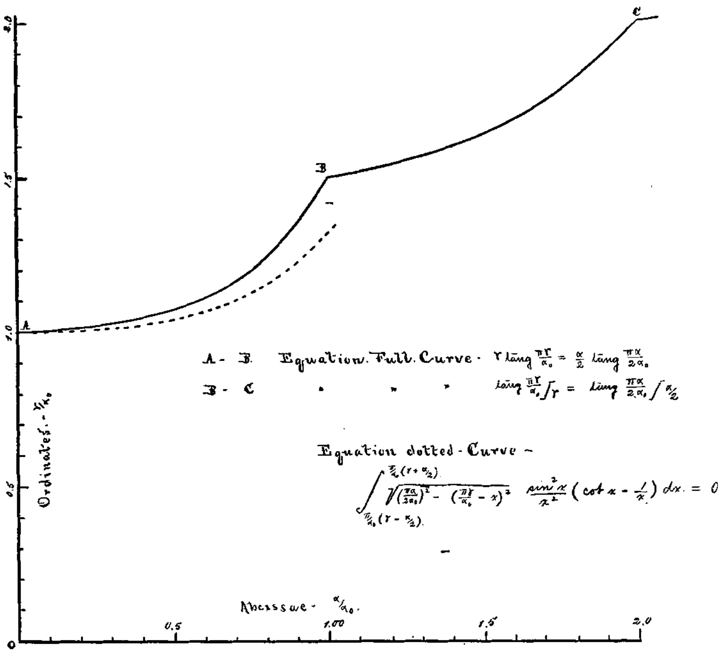
$$\tan\frac{\pi\alpha}{2\alpha_0}=\pm\infty, \quad \tan\frac{\pi\gamma}{\alpha_0}=\infty, \quad \text{and} \quad \frac{\gamma}{\alpha_0}=\frac{3}{2}, \frac{5}{2} \ldots \frac{2m-1}{2};$$

and when $\dfrac{\alpha}{\alpha_0}=2, 4, 6, \ldots 2m,$

$$\tan\frac{\pi\alpha}{2\alpha_0}=0, \quad \tan\frac{\pi\gamma}{\alpha_0}=0, \quad \text{and} \quad \frac{\gamma}{\alpha_0}=1, 2, 3, \ldots m.$$

The curve obtained is shown in fig. 9.

Fig. 9.



2nd. When the opening observed is circular, the expression for the intensity becomes

$$\mathrm{I}=\int_{-\alpha/2}^{+\alpha/2}\sqrt{\left(\frac{\alpha}{2}\right)^2-\phi^2}\,\frac{\sin^2\frac{\pi}{\alpha_0}(\gamma-\phi)}{\left[\frac{\pi}{\alpha_0}(\gamma-\phi)\right]^2}d\phi. \quad . \quad (24)$$

Differentiating with respect to γ,

$$\frac{d\text{I}}{d\gamma}=\text{C}\int_{-a/2}^{+a/2}\sqrt{\left(\frac{a}{2}\right)^2-\phi^2}\,\frac{\sin^2\frac{\pi}{\alpha_0}(\gamma-\phi)}{\left[\frac{\pi}{\alpha_0}(\gamma-\phi)\right]^2}\left(\cot\frac{\pi}{\alpha_0}(\gamma-\phi)-\frac{1}{\frac{\pi}{\alpha_0}(\gamma-\phi)}\right)d\phi.$$

Substituting $x=\frac{\pi}{\alpha_0}(\gamma-\phi)$, we have finally

$$\frac{d\text{I}}{d\gamma}=\int_{\pi/\alpha_0\,(\gamma-a/2)}^{\pi/\alpha_0\,(\gamma+a/2)}\sqrt{\left(\frac{\pi a}{2\alpha_0}\right)^2-\left(\frac{\pi}{\alpha_0}(\gamma-x)\right)^2}\,\frac{\sin^2 x}{x^2}\left(\cot x-\frac{1}{x}\right)dx=0. \qquad (25)$$

Integrating this for different values of $\frac{a}{\alpha_0}$ and $\frac{\gamma}{\alpha_0}$, we obtain the curve shown in fig. 9, which can be expressed very closely indeed for $0<\frac{a}{\alpha_0}<1$ by the simple formula

$$\left(\frac{\gamma}{\alpha_0}\right)_c=\frac{1}{\pi}\left(\pi+2\left[\left(\frac{\gamma}{\alpha_0}\right)_s-1\right]\right), \quad . \; . \; . \quad (26)$$

where

$\left(\frac{\gamma}{\alpha_0}\right)_c$ is the ordinate to the dotted curve,

$\left(\frac{\gamma}{\alpha_0}\right)_s$ „ „ full curve for any given value of $\frac{a}{\alpha_0}$.

The results of some observations by this method on the size of a slit and of circular openings of varying diameters, gave, in the case of the slit, errors varying from one to fifteen per cent., the average error being about eight per cent.; and in the case of circular openings errors of from three to twenty-five per cent., the average error being about twelve per cent. The accuracy of both this and the preceding method would undoubtedly be increased by taking the mean of a number of observations.

Of the two methods, the first is by far the most accurate; but even the second gives results which are from eight to ten times as accurate as those which can be obtained by using the telescope directly.

The apparatus by which the observations in the preceding tables were made is shown in Plate II. fig. 1. The objective of the telescope (a very fine four-inch glass, for the use of which I am indebted to the Worcester Polytechnic Institute) was fitted with a pair of adjustable slits, whose distance apart could be regulated by a right-and-left-hand screw geared to

Fig. 1.

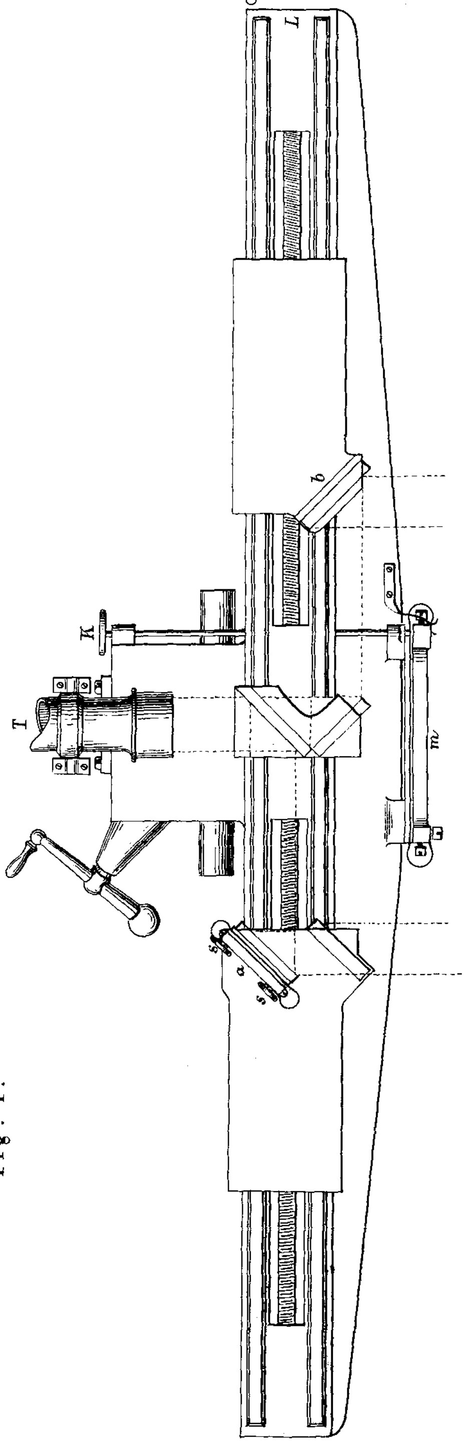


Fig. 2.

Mintern Bros. lith.

a rod (as shown in the Plate), so as to be controlled by the observer, and the distance between the slits was read off by a small telescope beside the eyepiece of the large one.

This simple and effective device has but one disadvantage. When the angle to be measured is so small as to be just beyond the power of the telescope, it is necessary, in order to observe the first disappearance of the fringes, that the slits should be separated to the full width of the aperture. Under these circumstances the two pencils meet at a very large angle, usually several degrees, and the corresponding distance between the inteference-fringes is but a few thousandths of a millimetre, and in order to be visible as such must be highly magnified and accordingly very faint.

This difficulty may be entirely overcome by using instead of the telescope one of the forms of refractometers shown in figs. 10 and 11.

Fig. 10.

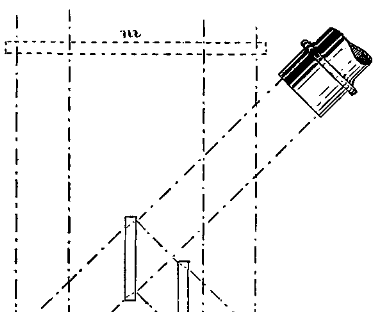
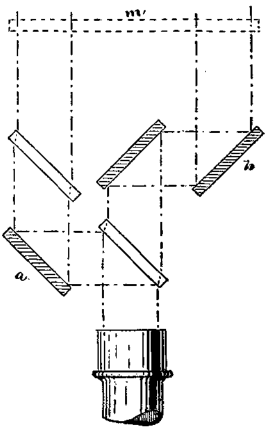


Fig. 11.



The first of these will be recognized as one of the forms already described *.

The apparatus being adjusted so that interference-fringes are visible, the telescope (a comparatively small one) is adjusted until the two images of its cross-hairs in the mirror *m* coincide with the cross-hairs themselves.

When this is the case, it is clear that when the whole apparatus is pointed at a star (the mirror *m* being now

* "Measurement by Light-Waves," by A. A. Michelson. American Journal of Science, xxxix. Feb. 1890.

removed) the two pencils will meet in the focus in the same phase, and the breadth of the interference-bands may be regulated by slight alterations of the pieces of the refractometer, *without in any way affecting the intensity of the light.* But beside this incidental advantage is the much more important one, that it is possible, by simply increasing the distance between the mirrors *a b*, to enormously increase the effective aperture of the telescope.

Plate II. fig. 2 shows, on a scale of one-fourth actual size, a plan of a proposed instrument for effecting this object. The optical arrangement adopted is that shown in fig. 11.

This form, notwithstanding the greater number of glasses, has some decided advantages over that of fig. 10. In addition to greater facility and ease of adjustment, it has the important advantage of preserving constant the width of the fringes notwithstanding the angular displacement of the whole apparatus *.

The instrument figured in Plate II. would be used as follows :—The mirrors *a* and *b* being moved as close together as possible, and the auxiliary mirror *m* being in place, the mirror *a* is adjusted by the screws SS till the two images of a source of light L as viewed in the telescope T appear to coincide. Next the mirror *m* is adjusted in azimuth by the screw K till the paths of the two pencils are equal. (As before mentioned, this angular motion does not affect the mutual inclination of the pencils, and therefore the breadth of the fringes is unaltered.) The telescope T is then adjusted till the two images of its illuminated cross-hairs coincide with the cross-hairs themselves, and is then clamped.

The mirror *m* is now detached and the instrument is ready for use.

Suppose the object to be measured is a minor planet or satellite. The whole instrument, which would have to be placed on an equatorial mounting, is pointed so that the image of the body is exactly on the cross-hairs. The interference-fringes will at once appear if the adjustment has been properly made.

Next, by means of a right-and-left-hand screw, the mirrors *a* and *b* are separated *until the fringes disappear.* If this disappearance is due to an accident, it will immediately become evident by observing any star in the neighbourhood. If the examination of the star shows the fringes while they

* The same advantage obtains in the use of the instrument as a refractomer for measuring extremely small variations of the angular position of the mirror or mirrors *m*.

are absent in the case of the planet, it may be considered certain that the cause of the disappearance in the latter case is its appreciable disk.

The angular diameter of the latter can be found (on the supposition of uniform illumination) by the formula

$$\alpha = 1{\cdot}22\,\frac{\lambda}{b},$$

where λ is the wave-length of the light, and b the distance between the centres of the mirrors a and b *.

From what has gone before, it will be inferred that the chief object of the method proposed is the measurement of the apparent size of minute telescopic objects, such as planetoids, satellites, and possibly star disks, and also double stars too close to be resolved in the most powerful telescope. But it is clear that interference methods may also be employed for the measurement of star-places.

Thus, in observations for right ascension, the slits would be placed parallel to the meridian, and the instant of passage of the central white fringe across the spider-lines noted ; and in observations for declination, the slits would be horizontal, the cross-hair being brought to coincide with the centre of the white fringe.

The increase in accuracy to be expected from this method would, however, be limited by the imperfections of our present means of measuring time and angles ; still, it would appear that by its use a one-inch glass may be made to do the work now required of a ten-inch.

Conclusion.

(1) Interference phenomena produced under appropriate conditions from light emanating from a source of finite magnitude become indistinct as the size increases, finally vanishing when the angle subtended by the source is equal to the smallest angle which an equivalent telescope can resolve, multiplied by a constant factor depending on the shape and distribution of light in the source and on the order of the disappearance.

(2) The vanishing of the fringes can ordinarily be determined with such accuracy that single readings give results from fifty to one hundred times as accurate as can be obtained with a telescope of equal aperture.

* Better, the distance between the centres of the apertures in front of these mirrors.

(3) The principal applications of the methods herein described are the measurement of the apparent magnitudes of very small or very distant sources of light such as planetoids and satellites (though larger bodies are not excluded), and of the angular distances between very close double stars.

(4) On account of the narrowness of the interference-fringes when a very minute body is under examination, the method of obtaining these fringes (by a pair of adjustable slits in front of the objective of a telescope) is open to objection, from which the refractometer method is entirely free. Further, this last modification makes it possible to extend the effective aperture of the equivalent telescope without limit. Thus, while it would be manifestly impracticable to construct objectives much larger than those at present in use, there is nothing to prevent increasing the distance between the two mirrors of the refractometer to even ten times this size. If among the nearer fixed stars there is any as large as our sun, it would subtend an angle of about one hundredth of a second of arc ; and the corresponding distance required to observe this small angle is ten metres, a distance which, while utterly out of question as regards the diameter of a telescope-objective, is still perfectly feasible with a refractometer. There is, however, no inherent improbability of stars presenting a much larger angle than this; and the possibility of gaining some positive knowledge of the real size of these distant luminaries would more than repay the time, care, and patience which it would be necessary to bestow on such a work.

In concluding, I wish to take this opportunity of expressing my appreciation of the disinterested manner in which my efforts have been so ably and zealously seconded by Mr. F. L. O. Wadsworth, Fellow of Clark University.

II. *On the Electrification of the Effluvia from Chemical or from Voltaic Reactions.* *By* J. Brown *.

1. FROM the series of very interesting experiments published in the January number of this Journal, p. 56, Mr. J. Enright concludes that nascent hydrogen and other gases become positively electrified by *contact* with acids, and negatively by *contact* with salts in solution. The line of experiment struck out by Mr. Enright will, I think, afford valuable aid in investigating electrochemical hypotheses ; but it would

* Communicated by the Secretaries of the Electrolysis Committee of the British Association.