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junctions are in use together, it is better to have a larger selector-switch with multiple ways, instead of using the four-way one provided on the potentiometer proper.

All terminals of the potentiometer are arranged in one row at the back, and a cover over the box is provided, which, however, does not come over the terminals, so that the wires need not be detached when the instrument is not in use.

I am indebted for suggestions and help to Mr. Keeling, and the making and adjustment of the coils of the instrument has been mainly done by Mr. Melsom, while the rest of the constructional work has been carried out very efficiently by the mechanic to the physical department, Mr. Murfitt.

Teddington, Feb. 1903.

VI. *On Refraction at a Cylindrical Surface.*

By ARTHUR WHITWELL, *M.A.*, *A.R.C.Sc.I.**

[Plate III.]

THE object of this paper is to describe and illustrate the position and form of the focal areas produced by the refraction at a cylindrical surface, bounding two media of different refractive indices, of light diverging from or converging to a point.

In general, when light diverging from or converging to a point falls on any surface bounding two media of different refractive indices, if a plane can be drawn through the point to cut an element of, or the whole surface, symmetrically, then all the light will really or virtually pass through a focal line or focal area in this plane. It is usual, when considering elements of the surface only, to use the term focal line, but it should always be remembered that these focal lines are in general elements of area. In the case we are about to consider there are two planes, about which the surface is symmetrical, which can be drawn through the radiant-point. One of these planes will contain the axis of the cylinder, and the other will be normal thereto.

We will consider first the plane containing the radiant-point and the axis of the cylinder.

Let fig. 1 (Pl. III.) be a plan and elevation of the cylinder of radius r, and let the radiant-point o be at a distance a from the axis. Draw the elevations and plans of two symmetrical incident rays and of the corresponding normals and refracted rays, join $o\,c$ and produce to meet the refracted ray in a point g. The two refracted rays will meet at the point g,

* Communicated by the Physical Society: read March 27, 1903.

and the locus of the point g will be the focal line. A plane containing the points o, a, c will contain an incident ray and the normal, and will therefore also contain the corresponding refracted ray. This plane will cut the plane of the figure (1) in the line ocg, and the refracted ray will in general cut this line ocg in some point g. Again, a plane containing the points o, b, c will contain the corresponding symmetrical incident ray, normal, and refracted ray, and will cut the plane of the figure in the line ocg. From considerations of symmetry it follows that the two refracted rays will intersect in the same point g on the line $o\,c\,g$. If we consider only a thin slice of the cylinder parallel to the plane of the paper in fig. 1, the elevations of the lines oa, ac, and ag may be taken as their true lengths.

Let the vertical aperture $= h$,
The angle of incidence $= \theta$,
" of refraction $= \phi$,
" agc $= \beta$,
" com $= \psi$,
and the length cg $= d$.

Then we have

$$\beta = \psi - \phi$$

$$\sin\theta = \mu \sin\phi$$

$$d \sin\beta = r \sin\phi$$

$$d = \frac{r\sqrt{a^2+h^2}}{h\cot\phi - a},$$

from which we get

$$d = \frac{r\sqrt{a^2+h^2}}{\sqrt{\mu^2(a-r)^2+(\mu^2-1)h^2} - a}. \quad . \quad . \quad (1)$$

Taking the radiant-point as origin, the equation to the locus is

$$x = a + d\cos\psi,$$

or

$$x = a + \frac{ra}{\sqrt{\mu^2(a-r)^2+(\mu^2-1)h^2} - a},$$

and putting in the value of

$$h = \frac{ay}{x},$$

we get finally

$$y^2 = \frac{x^2}{a^2(\mu^2-1)}\left\{\frac{(ax+ra-a^2)^2-(\mu(a-r)(x-a))^2}{(x-a)^2}\right\}. \quad (2)$$

It may be stated here that the equation represents not only the locus of the intersections of the real refracted rays, but also of the false refracted rays. These false rays are equally inclined to but on the opposite side of the normal to the real rays. This arises from the fact that we have to square to get rid of the radical sign in the denominator of the expression for d; for this reason it is immaterial, as far as the equation arrived at is concerned, whether we take the + or − sign before the radical quantity in the denominator of d.

The curves represented by this equation, taking $\mu=\frac{3}{2}$ and $r=2$ and various values of a, are shown plotted in fig. 4, the light proceeding from left to right. When the radiant-point is at infinity, the focal line will be the straight line I. at a distance $\frac{\mu r}{\mu-1}$ from the surface, or $\frac{r}{\mu-1}$ from the axis of the cylinder. As the radiant-point moves up to the surface, the focal line gradually bulges out to the right at its centre, and its upper and lower ends bend towards and become asymptotic to the axis as shown by curve II., for which $a=10$. When the distance of the radiant-point from the surface $=\frac{r}{\mu-1}$, or when $a=\frac{\mu r}{\mu-1}$, the centre of the focal line breaks, and its two ends become parallel to, but at an infinite distance from, the axis of x. The curve III. has parabolic asymptotes, the equation of which, referred to the radiant-point as origin, is

$$y^2=3\cdot 2\,x+22\cdot 4.$$

This point, $a=\frac{\mu r}{\mu-1}$, will be recognized as the principal focus for light proceeding from right to left. When the radiant-point is inside this focus, the curve, IV., has two branches and a pair of rectilineal asymptotes, the axis of the cylinder remaining an asymptote. The branch on the left is of course virtual.

As the radiant-point moves to the right, the angle which the rectilineal asymptote makes with the axis of x increases from zero to a maximum, and then diminishes to zero. When the radiant-point is on the surface there is no focal line; an incident cone of light produces a refracted cone, the ratio of the sines of the semi-angles of the cones being $=\mu$.

I have not plotted the false focal lines in curves I.–IV.; they all lie between the surface of the cylinder and the axis, to which they are asymptotic.

When a lies between r and $\frac{\mu r}{\mu+1}$, the real focal line lies between the surface and axis of the cylinder, and the false focal line has two branches and a pair of rectilineal asymptotes. When $a=\frac{\mu r}{\mu+1}$ the real line is shown by curve V., and the false line has parabolic asymptotes, the equation of which is

$$y^2=3\cdot 2x+7.$$

A graphical method of drawing the asymptotes may be obtained by considering equation (1),

$$d=\frac{r\sqrt{a^2+h^2}}{\sqrt{\mu^2(a-r)^2+(\mu^2-1)h^2}-a}.$$

If $\mu^2(a-r)^2+(\mu^2-1)h^2=a^2$, d is infinite, and this relation between a and h gives us the circle shown in fig. 4.

To obtain the asymptotes, draw an ordinate to the circle through the radiant-point, and project horizontally to a point on the axis of the cylinder. The asymptote will pass through this point on the axis, and through the radiant-point. When $a=0$ the focal line will be the axis itself, and this is the only case in which the focal line will be a mathematical straight line whether the aperture be large or small.

When a becomes negative the curve is still asymptotic to the axis (see curve VI., for which $a=-10$), and as a increases the curve gradually moves to the right and approaches the line I., which it reaches when a is infinite.

The set of curves shown in fig. 5 are for the case in which light proceeds from a denser to a rarer medium, and are obtained by putting $\mu=\frac{2}{3}$ in equation (2).

When the radiant-point is at infinity the focal line is a straight line, VII., at a distance $\frac{r}{\mu-1}$ from the surface, or $\frac{\mu r}{\mu-1}$ from the axis: it is virtual. As the radiant-point moves to the right, the curve becomes of the form shown by VIII., for which $a=10$. In this curve, when the vertical aperture $h=\pm\frac{\mu(a-r)}{\sqrt{1-\mu^2}}$, total reflexion occurs, and the focal line cuts the surface; the continuation of the curve inside the cylinder is the false focal line.

As a diminishes, the curve becomes smaller (curve IX. is for $a=6$), and finally diminishes to a point when $a=r$; in this case a cone of light having a semi-vertical angle equal to the critical angle is refracted, the remainder of the incident light being totally reflected, and the refracted cone having a semi-angle of $\frac{\pi}{2}$. When the radiant-point is inside the cylinder, or when the light is convergent, the curve becomes of the form X., for which $a=1$. In this case the part of the curve inside the cylinder is the real, and the part outside the false focal line.

When $a=\frac{\mu r}{\mu+1}$ the curve, XI., has parabolic asymptotes, the equation of which is

$$y^2=-7\cdot 2\,x-13.$$

When a lies between $\frac{\mu r}{\mu+1}$ and $\frac{\mu r}{\mu-1}$ the curve has two branches and a pair of rectilineal asymptotes. Curve XII. is for $a=\cdot 5$, and the branch on the right and that part of the branch on the left which is outside the cylinder are the false focal line, the real focal line being that part of the left branch which is inside the cylinder. The asymptotes may be drawn by means of a circle, the equation of which is obtained as before.

When $a=0$, the axis is the focal line. When a lies between 0 and $\frac{\mu r}{\mu-1}$ or -4, the curve has two branches; the branch on the right and that part of the branch on the left which is outside the cylinder are the real focal line, and that part of the branch on the left which is inside the surface is the false focal line.

When $a=-4$, the curve, XIII., has parabolic asymptotes, the equation of which is

$$y^2=-7\cdot 2x+21,$$

this point being the principal focus for rays passing from right to left.

When a is negative and greater than 4 the curve has only one branch, which lies on the left of the axis, that part outside the surface being the real and that part inside the false focal line. Curve XIV. is for $a=-10$.

As the radiant-point moves off to infinity towards the right, the curve approaches and ultimately coincides with the straight line VII.

The curves shown in figs. 4 and 5 are for light falling on the convex surface of the cylinder, but by reversing them about the axis of the cylinder they will represent the focal lines produced by light passing from left to right and falling on the concave surface of the cylinder if the corresponding values of a be also reversed in sign.

We shall now find the equation of the locus of the intersection of two symmetrical rays which have the greatest angle of incidence possible, viz. $\frac{\pi}{2}$. Suppose the triangle oag, fig. 1, to be turned down into the plane of the paper on the line og, as shown in fig. 2. We know that

$$oc = \sqrt{a^2+h^2}\,; \quad oa = \sqrt{a^2-r^2+h^2}\,; \quad \theta = \frac{\pi}{2}\,;$$

and that
$$\sin\phi = \frac{1}{\mu}.$$

We also have
$$r\sin\phi = d\sin\beta,$$

$$d = \frac{r}{\mu\sin\beta} = \frac{r\sqrt{a^2+h^2}}{\sqrt{\mu^2-1}\sqrt{a^2-r^2+h^2}-r}. \quad . \quad (3)$$

Taking the radiant-point as origin, $x = a + d\cos\psi$, or

$$x = a + \frac{ra}{\sqrt{\mu^2-1}\sqrt{a^2-r^2+h^2}-r},$$

and putting $h = \frac{ay}{x}$, the equation reduces to

$$y^2 = \frac{x^2}{a^2(\mu^2-1)}\left\{\frac{x^2r^2-(\mu^2-1)(a^2-r^2)(x-a)^2}{(x-a)^2}\right\} \quad (4)$$

Putting $\mu = \frac{3}{2}$, $r=2$, the locus represented by this equation is shown plotted in fig. 6. The general nature of the curves is the same as that of the curves for small apertures shown by fig. 4. As before, we can get a relation between a and h for which the corresponding value of d is infinite.

In this case $d=\infty$ when $a^2+h^2 = \frac{\mu^2r^2}{\mu^2-1}$; this relation represents a circle of radius $= \frac{\mu r}{\sqrt{\mu^2-1}}$.

When $a=\infty$ the focal line is a straight line, XV., at a distance $\frac{r}{\sqrt{\mu^2-1}}$ or 1·78 from the axis. As the radiant-point moves to the right, the curve bulges out at the centre and bends towards the axis at each end, as shown by curves XVI. and XVII., for which $a=10$ and $a=4$.

When $a=\frac{\mu r}{\sqrt{\mu^2-1}}=2{\cdot}68$, the principal focus for rays going from right to left, the curve, XVIII., has parabolic asymptotes.

When a lies between $\frac{\mu r}{\sqrt{\mu^2-1}}$ and r, or between $\frac{-\mu r}{\sqrt{\mu^2-1}}$ and $-r$, the curve has three branches and a pair of rectilineal asymptotes. Curve XIX. is for $a=2{\cdot}4$; the false focal line is not drawn, and the branch on the left is virtual.

When a lies between r and $-r$, we cannot have an angle of incidence of $\frac{\pi}{2}$.

When a lies between $-r$ and $-\frac{\mu r}{\sqrt{\mu^2-1}}$ we get a curve like XIX., but reversed about the axis of y. As the radiant-point moves to the right, the focal line gradually approaches and finally coincides with the line XV. (See curve XX., which is for $a=-10$.)

When the light passes from a denser to a rarer medium we cannot obtain the equation of the locus of the intersection of symmetrical refracted rays by putting μ less than unity in equation (4), because for an angle of incidence of $\frac{\pi}{2}$ the rays would be totally reflected. The maximum angle of incidence will be the critical angle; we shall, therefore, find the equation of the locus for symmetrical rays having an angle of incidence the sine of which $=\frac{2}{3}$. When the triangle *oag* is folded down into the vertical plane on the line *og*, we shall get the construction shown in fig. 3, in which θ is the angle of incidence, ϕ the angle of refraction. We have

$$\sin\theta=\mu\,;\ \beta=\theta-\alpha\,;\ r\sin\theta=\sqrt{a^2+h^2}\sin\alpha\,;\ d=\frac{r}{\cos\beta},$$

or

$$d=\frac{r\sqrt{a^2+h^2}}{\sqrt{1-\mu^2}\sqrt{a^2+h^2-\mu^2r^2}+\mu^2r}\quad . \quad . \quad . \quad (5)$$

$$x=a-d\cos\psi,$$

and putting $h=\frac{ay}{x}$ we get finally

$$y^2=\frac{x^2}{a^2(1-\mu^2)}\left[\frac{(\mu^2ra-ra-\mu^2rx)^2-\{(x-a)^2(1-\mu^2)(a^2-\mu^2r^2)\}}{(x-a)^2}\right].$$

The locus represented by this equation is shown plotted in fig. 7, r being $=2$ and $\mu=\frac{2}{3}$.

The equation to the circle which gives the asymptotes is

$$a^2+h^2=\frac{\mu^2r^2}{1-\mu^2}.$$

When $a=\infty$ the focal line is the straight line XXI. at a distance $\frac{r}{\sqrt{1-\mu^2}}$ or 2·68 from the axis. As the radiant-point moves to the right the focal line also moves to the right and becomes asymptotic to the axis, as shown by curve XXII. for which $a=10$. The branch of the curve XXII. on the right is the focal line for light falling on the concave surface. The real and false focal lines coincide since the refracted ray is at right angles to the normal at the point of incidence. The parts of the curves inside the cylinder have no real existence, they are the loci of intersection supposing that total reflexion did not occur. As the radiant-point approaches the surface the real portion of the focal line becomes shorter and dwindles to zero when the radiant-point reaches the surface. Curve XXIII. is for $a=\frac{\mu r}{\sqrt{1-\mu^2}}$ or 1·78, which is the point where the asymptote circle cuts the axis of x; the branch on the left, of which only a minute portion is real, is the focal line for light falling on the convex surface of the cylinder, and the branch on the right, which has parabolic asymptotes, is the focal line for light falling on the concave surface of the cylinder.

Curve XXIV. is for $a=1·33$. The branch on the right and the left-hand part of the branch on the left above the axis of x is the focal line for light falling on the concave part of the cylinder, whilst the remainder of the left-hand branch is the focal line for light falling on the convex portion of the cylinder. When $a=0$ the axis is the focal line, when a is negative the curves are got by reversing the corresponding curves for positive values of a. Thus curve XXV., which is curve XXII. reversed, is for $a=-10$.

As the radiant-point moves off to the right the focal line gradually approaches and ultimately coincides with the line XXI. from which we started.

We have now plotted the focal lines for maximum and minimum horizontal aperture in all possible cases. A curve from fig. 4 or fig. 5 and the corresponding curve from fig. 6 or fig. 7, *e.g.* curves IV. and XVII., will define the focal area, that is to say, all the light from the corresponding radiant-point will after refraction pass through the area between these curves. If we suppose ourselves at the radiant-point and facing the cylinder half the light will be bent from left to right and the other half from right to left, but it will all pass through the central plane containing the radiant-point and the axis, in the area between these two curves. It is easily seen that the width of the focal area on the axis of x will be equal to the spherical aberration of the section of the cylinder made by a horizontal plane containing the axis of x.

We will now consider the focal areas produced in the second symmetrical plane, viz.: the plane containing the radiant-point and normal to the axis of the cylinder. Draw a similar construction to that shown in fig. 1 for two rays symmetrical with regard to the horizontal plane, and produce the refracted rays backwards till they intersect at the point k in the horizontal plane. This point k will be on a straight line drawn through the radiant-point and parallel to the two normals.

Consider, first, a thin horizontal slice of the cylinder containing the axis of x. For small horizontal aperture the distance $cf=\frac{ra}{(\mu-1)a-\mu r}$. By small apertures I mean those for which one can neglect the spherical aberration in comparison with the length cf.

Let

$$cf=c, \quad \text{and } ok=d'.$$

Then from the figure we have

$$\frac{c}{a+c}=\frac{r}{d'}$$

$$d'=\frac{r(a+c)}{c}=(\mu-1)(a-r).$$

For small horizontal apertures then the focal line is an arc of a circle having its centre at the radiant-point and its radius $=(\mu-1)$ times the distance of the radiant-point from the surface. The focal line is virtual for diverging light and real for converging light. If we take two horizontal strips of the cylinder at a distance of h above and below the horizontal plane the focal line formed by rays which fall on these two strips will also be a circular arc for small horizontal apertures. Its radius d'' is obtained from the relation

$$\frac{d}{\sqrt{a^2+h^2}}=\frac{r}{d''}$$

$$d''=\frac{r\sqrt{a^2+h^2}}{d}=\sqrt{\mu^2(a-r)^2+(\mu^2-1)h^2}-a.$$

All the light which falls on the cylinder will therefore virtually pass through the area in the horizontal plane between arcs of two circles of radius d' and d'' having the radiant-point as centre. The width of the focal area on the axis of $x = d'' - d'$ and this width increases as the vertical aperture h increases, and becomes infinite when h is infinite.

The focal lines or areas we have discussed are produced by the intersections of symmetrical rays. Besides these the refracted rays produce two caustics which are the loci of intersections of consecutive rays. The first is the locus of the intersections of consecutive refracted rays in the plane containing the radiant-point and the axis of the cylinder, and is the same as that which would be produced by rays in one plane refracted at a plane surface dividing any two media of different refractive indices. The second is the locus of the intersections of consecutive refracted rays in the plane containing the radiant-point and normal to the axis of the cylinder, and is the caustic of the circle. Incident rays in oblique sections of the cylinder do not produce caustics since the corresponding refracted rays are not in the same plane.

If we suppose light from a radiant-point to fall on a cylinder the radius of which gradually increases to infinity, it will be seen that ultimately, when the curvature is zero, the two focal areas will coincide and be reduced to a short piece of the line through the radiant-point and normal to the surface. Similarly, if we suppose the cylinder to become a semi-ellipsoid which gradually becomes a hemisphere, then the two focal areas will ultimately coincide and become a portion of the line joining the radiant-point and the centre of the hemisphere.

Instead of having caustics in two planes only we now have a caustic in every plane passing through the radiant-point and the centre of the hemisphere.

If light proceeding from or to a point a fall on the plane or spherical surface of a plano- or sphero-cylindrical lens of small aperture it will pass on to the cylindrical surface as if it proceeded from a point a', a and a' being conjugate with respect to the first surface. The focal areas produced by a plano- or sphero-cylindrical lens are therefore, for small apertures, identical with those produced by refraction at a single cylindrical surface; and if we define two optical systems as "equivalent" when they produce identical focal areas, then we can say that a plano- or sphero-cylindrical lens with the radiant-point at a is equivalent to a single cylindrical surface with the radiant-point at a'; a and a' being conjugate with respect to the first surface.

In conclusion, I should like to call attention to a misleading statement made by Prof. S. P. Thompson, in a paper on this subject read before this Society on December 8th, 1899 *. He says: " In any lens having at one surface a radius of curvature r, the curvature which that surface will impress upon a plane wave is $\frac{\mu-1}{r}$; where μ is the refractive index of the material. If the lens is cylindrical, having a curvature in one meridian only, the impressed curvature will also be cylindrical."

"Let AA′ be the axis of a cylindrical lens, and NN′ a line normal to that axis. A plane normal to the axis intersecting the lens in NN′ will have as its trace through the curved surface of the lens a line of the same curvature as the lens, viz. $\frac{1}{r}$. Let now an oblique intersecting plane be drawn through the optic axis of the system; its intersection PP′ making an angle $NOP=\phi$ with the line NN′. The curvature at O of the trace of this plane, where it cuts the curved surface along PP′, will be $\frac{1}{r}\cos^2\phi$.... We may further consider the intersection QQ′ of another oblique plane at right-angles to PP′. The curvature at O along the line QQ′ will be $\frac{1}{r}\sin^2\phi$.... If light were admitted through narrow parallel slits set respectively along PP′ and QQ′, the convergivity of the two beams respectively impressed by the lens would be $(\mu-1)\frac{\cos^2}{r}\phi$ and $(\mu-1)\frac{\sin^2}{r}\phi$. If r is expressed in metres, then these two convergivities will be expressed in dioptries...."

Now if a plane wave fall on a thin plano-cylindrical lens the emergent wave-surface, for small aperture, will be a cylinder of radius $\frac{r}{\mu-1}$. Every refracted ray will pass through a line or narrow band parallel to the axis of the cylinder and at a distance $\frac{r}{\mu-1}$ from the lens. If we suppose a card with a narrow diagonal slit to be placed in front of the lens it is obvious that a great part of the cylindrical wave-surface will be cut off, but the portion that remains will still be cylindrical, and will have the same radius. The rays that pass through the slit still pass through the line at the distance $\frac{r}{\mu-1}$, and the convergivity is the same as before.

* Phil. Mag. March 1900.

The power of impressing convergence on a plane wave of the diagonal strip of the lens is the same as the power of the whole surface. A cylindrical lens can only properly be held to have *two* powers, and it appears to me to be a mistake to speak of the power of a cylindrical lens along a line making an angle ϕ with the axis as being $\frac{\mu-1}{r}\sin^2\phi$.

If parallel light be allowed to fall on a cylindrical lens and a ground-glass screen be placed at a distance of $\frac{r}{\mu-1}$ from the lens, then a line of images of the source of light, parallel to the axis of the cylinder, is seen on the screen. If a stop with a diagonal slit be now inserted and the screen be moved up close to the lens, a line of images out of focus, less intense than before, and parallel to the slit is seen on the screen; as the screen is moved away this line of images rotates and gradually becomes sharper, until when at length the screen reaches the distance $\frac{r}{\mu-1}$ the line of images is parallel to the axis of the cylinder and is perfectly sharp. This shows that the slit has made no difference to the position of the focus or to the power of the lens. As the screen moves further back still, the line of images continues to rotate in the same direction and gets more and more fuzzy or out of focus.

In a paper entitled "On Astigmatic Lenses," read before this Society on November 9th, 1900 *, Mr. R. J. Sowter makes the same mistake, and speaks of the power of a cylindrical lens in a direction OR making an angle ϕ with the axis of the lens as $A\sin^2\phi$, where A is "the equatorial or focal power of the lens." He also speaks of the power of a plano-ellipsoidal lens along a direction OR making an angle ϕ with an axis of the elliptic plane face of the lens as being $=A\cos^2\phi + B\sin^2\phi$, A and B being the two powers of the lens.

My remarks apply as well to an ellipsoidal lens as to a cylindrical. When the radiant-point is on an axis of an ellipsoidal lens, the light produces two focal areas and two caustics in the planes of maximum and minimum curvature. A thin slice of the lens parallel to the direction OR will not produce caustics, and the rays which pass through it will all pass through the same focal areas as they would if the whole lens were employed. A screen placed at distances $\frac{1}{A}$ or $\frac{1}{B}$ would show the same sharp but less intense lines of images of the source of light as when the whole lens was used.

* Phil. Mag. Feb. 1901.

Both of these writers appear to have considered the curvature of the section of the lens in any direction to be the important element in determining the position of the focal lines instead of the principle of symmetry.

I have to thank Prof. Everett for suggesting to me the reason why my equations represent the false focal lines as well as the real; also Mr. Lyndon Bolton for tracing the curve represented by equation (2), and showing that in a particular case it has parabolic asymptotes.

VII. *Spectra of Gases and Metals at High Temperatures.* *By* John Trowbridge*.

[Plate IV.]

The spectra of metals in atmospheric air are the visible evidence of extremely complicated chemical reactions due to the metallic vapour and the gases of the atmosphere. The spectra of gases also in narrow containing vessels of glass or of quartz are modified by the walls of these vessels when the temperatures of the gases are very high; moreover, the ordinary method of obtaining photographic spectra either of metals in air or rarefied gases, by long continued discharges produced by the Ruhmkorf coil or transformers, masks certain fundamental reactions.

It is therefore desirable to study the effect of known quantities of energy successively applied to produce spectra either of metals or gases. This can best be accomplished by charging a condenser to a known amount by a known electromotive force, and by discharging the condenser between terminals of metals either in air or in gases. If the spectra produced in this manner, by discharges varying from one to any desired number, are photographed on the same plate and treated alike in the same developer, the ground may be prepared for some generalization of the extremely complicated reactions I have mentioned. I believe that this method is a fundamental one to use if order is to be brought out of the chaos of spark-spectra.

I have applied this method in the following manner:—A storage-battery of from ten thousand to twenty thousand cells is employed to charge a condenser ·1 to ·3 microfarad. By a simple mechanical appliance the condenser is detached from the poles of the battery, and is discharged between suitable terminals. Although it is impossible to avoid a slight spark

* Communicated by the Author.

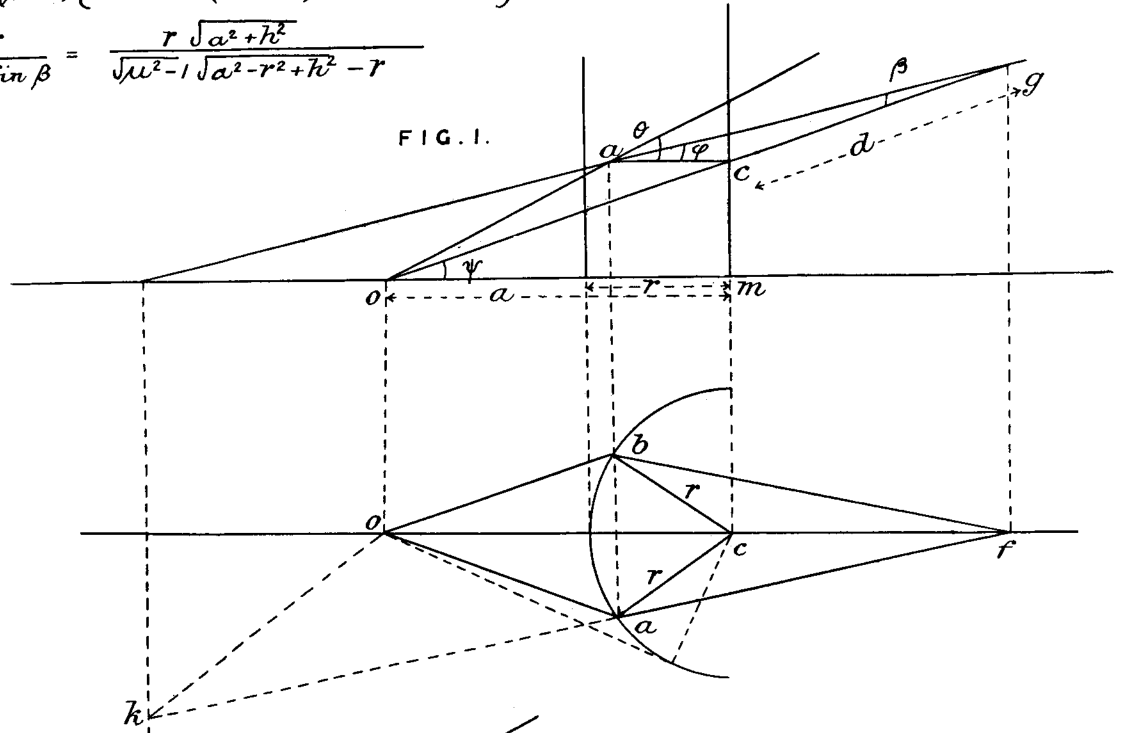
$$d = \frac{r\sqrt{a^2+h^2}}{\sqrt{\mu^2(a-r)^2+(\mu^2-1)h^2}-a}$$

$$x = a + d\cos\psi$$

$$y^2 = \frac{x^2}{a^2(\mu^2-1)}\left\{\frac{(ax+ra-a^2)^2-\mu(a-r)(x-a)^2}{(x-a)^2}\right\}$$

$$d = \frac{r}{\mu\sin\beta} = \frac{r\sqrt{a^2+h^2}}{\sqrt{\mu^2-1}\sqrt{a^2-r^2+h^2}-r}$$

FIG. 1.



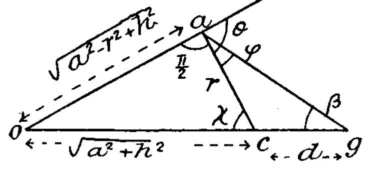


FIG. 2.

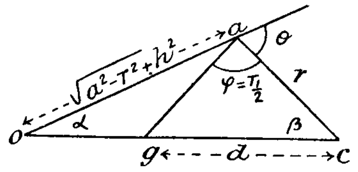


FIG. 3

FIG. 4.

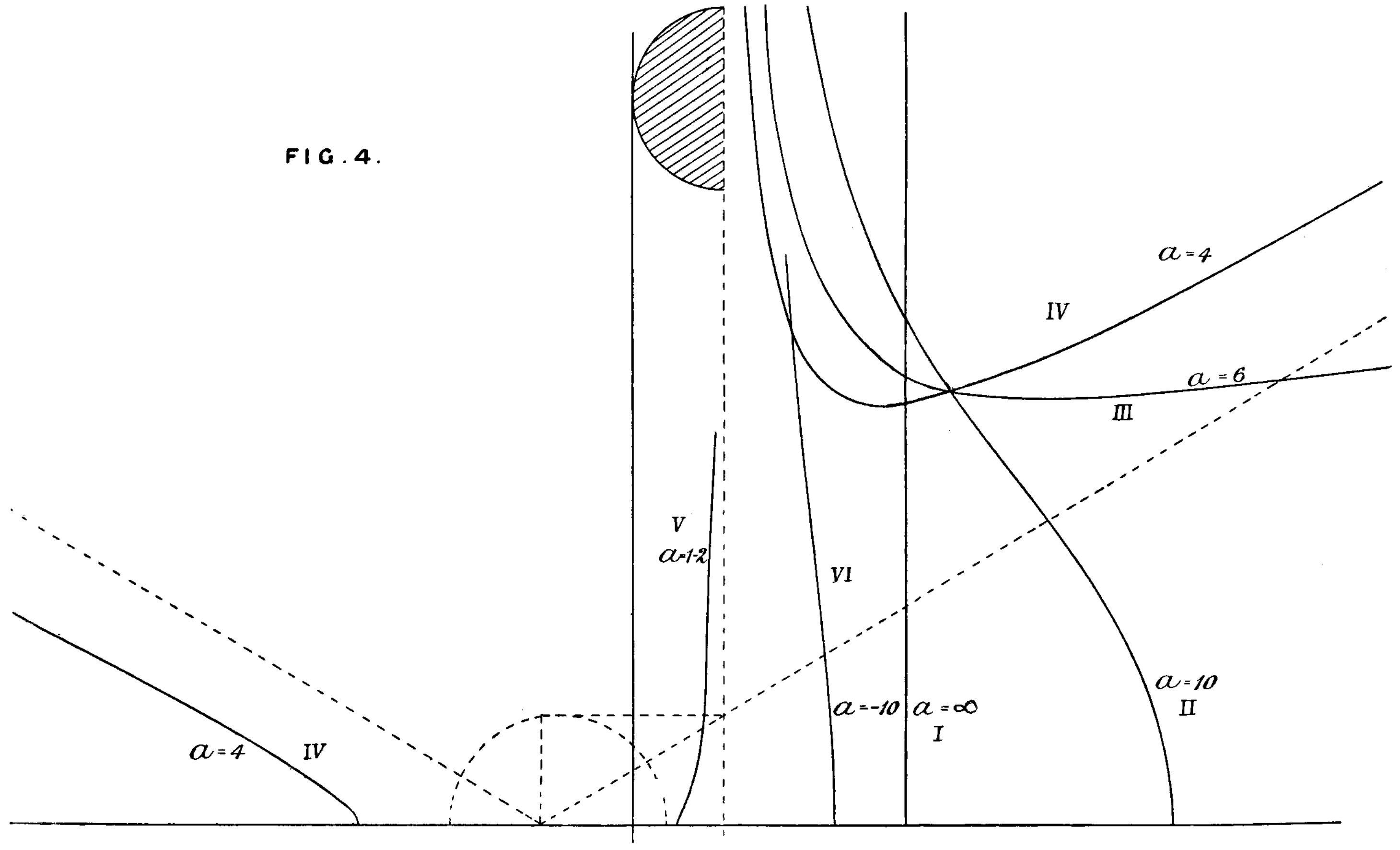


FIG. 5.

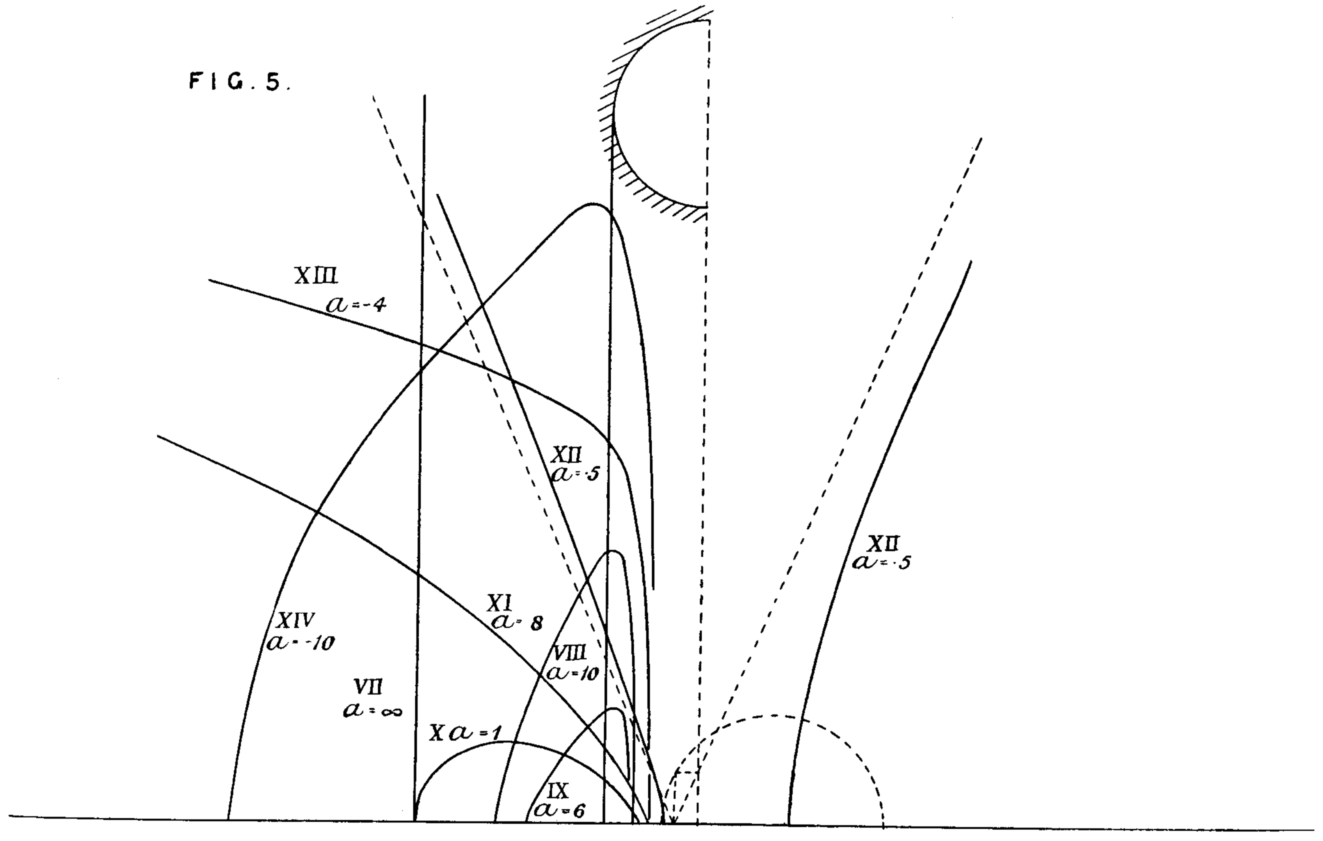


FIG. 6.

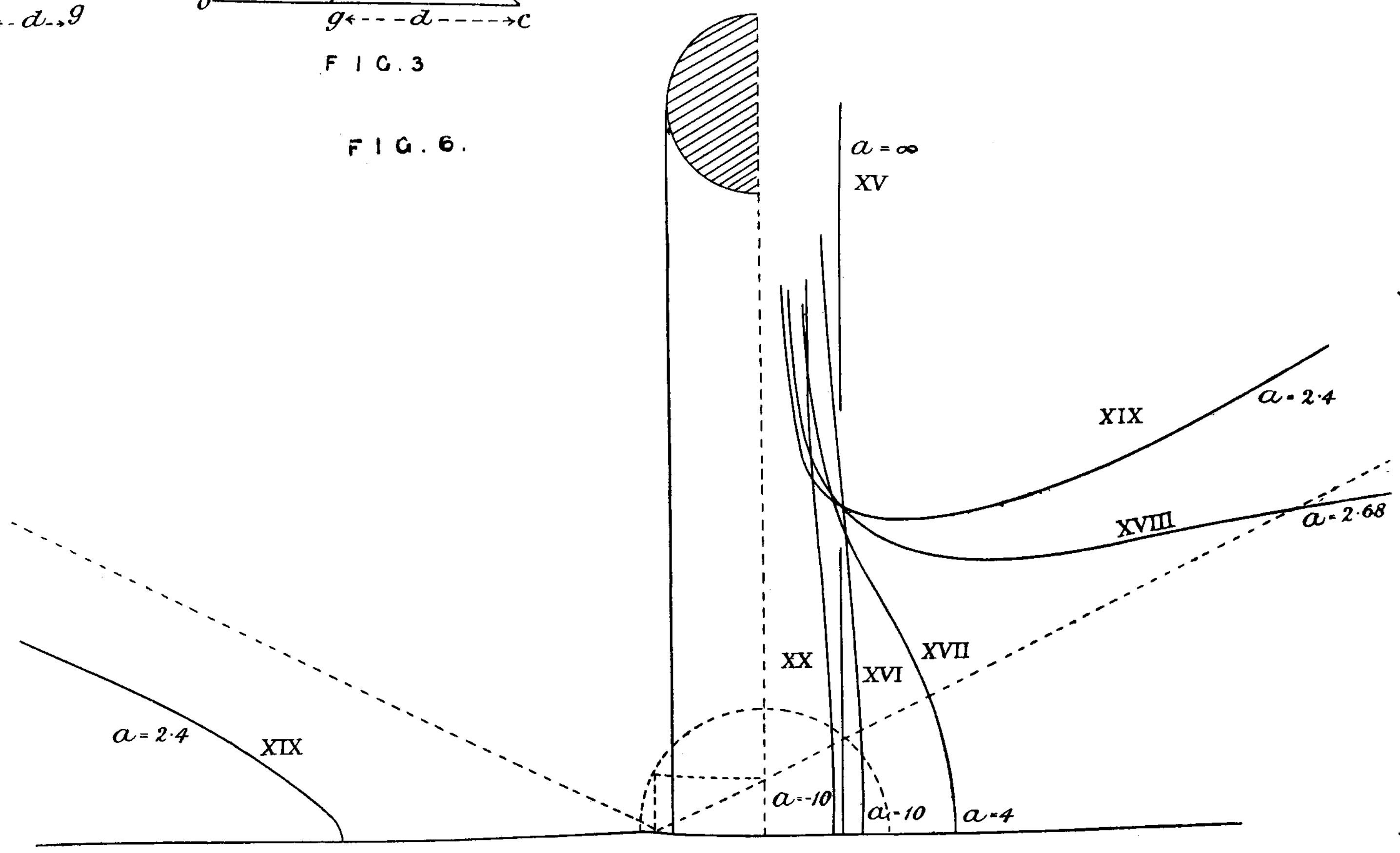


FIG. 7.