THE OBSTRUCTION TO THE NAVIGATION OF RIVERS CAUSED BY THE PIERS OF BRIDGES.

BY J. W. SPRAGUE.

The investigation, which was interrupted at the close of my last communication, as to the amount of power required for the ascent of a steamboat of specified dimensions through a given draw, was based upon the supposition that, so far as vertical resistances were concerned, all the power of the engine was usefully employed in overcoming the resistances; that there was no loss of power between the point where it was derived from the engine, and the point where it was applied to overcome the resistances. In truth this is far otherwise. Let us look at the causes of loss of power in its transmission. Those that I will consider arise from three sources—the *friction* of the wheel, the *obliquity* with which the paddle boards (in the case of the ordinary radial wheel) strike the water, and the *slip* of the wheel. Of these three causes of loss of power, the operation of the first two, in case of a steamboat ascending a draw, is similar to their operation in case of a steamboat moving in an unobstructed water-way. The operation of the *slip* is quite different. The first two may then be disposed of by assuming that, of the entire power given up by the engine, $\frac{1}{8}$th is consumed by them.

As regards the introduction of specific values, I would state that my object is to show *how* the values are to be used, not *what values* are to be used; and while in each case the values given are based upon the best data in my possession, yet I wish distinctly to disclaim for them great accuracy. The careful reader will readily distinguish between those values which are liable to variation, and those which are not; for instance, when it is stated that, to lift a boat of specified weight up a *remou* of definite hight requires the expenditure of a given amount of useful effect, there is no liability to error; but when we go further, and state that to accomplish this useful effect requires an expenditure of a particular amount of power at the engine, then we enter the region of uncertainty, the values varying with the medium through which the power is transmitted.

In order to determine the loss of power occasioned by the slip of the wheel when in the draw, it will be necessary to analyze the causes producing the slip. Take the case of a locomotive drawing a train of cars, at a definite uniform velocity, on a level dry track, where the adhesion of the driving wheel to the rail is perfect, and there is no slipping. The measure of the power expended, in going a specified distance, is the product of the equivalent mean pressure on the piston, multiplied by the distance passed over by the piston. The useful effect produced is the product of the constant tractive power required to draw such a train at such a velocity, multiplied by the actual distance passed over. Neglecting the loss of power within the engine itself by friction, &c., these two products will be equal, and the power expended will equal the useful effect produced. Again, suppose the same train to maintain the same velocity over another equal portion of *oiled* track, where the adhesion is not perfect. The useful effect produced will be the same as in the first case, for the oiling of the track may be regarded as having no effect on the tractive power required to move the train; but in order to maintain the same tractive power, on account of the *slip* of the wheel, it must revolve faster than in the first case, which implies an increased velocity of piston, and expenditure of power, without any increase of the useful effect produced. If, while the train advances three feet, the wheel rolls over four feet, the *slip* of the wheel will be one foot, or one-third of the velocity of the train: and as four parts of power are required to produce three parts of useful effect, one-fourth of the power expended is is wasted, or we must add one-third to the useful effect, in order to obtain the power to be expended for its production.

Just so is it in the case of a steamboat. If the wheel revolved with a velocity equal to the velocity of the boat, the paddles would quietly enter the water without striking against it; but as water is not a solid, but a yielding fluid, it is necessary that, in order to push the boat forward, the paddles must strike the water with an increased velocity. The increased velocity of the paddle wheel, requisite to convert the yielding water into a resisting medium, corresponds to the increased velocity of the driving wheel of the locomotive, requisite to maintain the necessary adhesion to the track. All power expended in overcoming the *slip* is lost in both cases. The amount of *slip* in the case of a steamboat will vary with different velocities, but for our purpose we may take the proportions given above, in the illustration of the locomotive, viz., one-third of the velocity of the boat. This value is, however, based upon the supposition that the velocity of the boat is *uniform*, and that the only resistances are those due to the boat's own motion, at any given velocity in *level* water.

Suppose, however, a steamboat has another boat in tow, then it is evident that to maintain any speed, the paddles must strike the water with more velocity than would be requisite for the same speed, if there was none tow—but the *slip* measures the velocity with which the paddles strike the water, and consequently in such a case the *slip* will be materially increased. Hence, whenever any steamboat encounters a resistance greater than that which is due to the boat itself moving at its present speed, the per-centage of slip will be increased. When a boat is increasing its speed, its resistance is measured, not merely by its present speed, but to this must be added the resistance due to the effort to increase the speed. When a steamboat is moving uniformly, at the rate of five miles per hour, the *slip* is less than of the same boat increasing its speed from four to six miles per hour, at the instant when it reaches five miles per hour.

When a steamboat ascends a draw, it encounters greater resistance than that due to the velocity of the current added to its own velocity; hence, for the reasons just given, the relative loss of power by *slip* is materially increased. In view of what has been said, I have little doubt that in case of a steamboat ascending a draw, the loss of power from the three causes alluded to amounts to one-half of the whole power.

In determining the power required to overcome the horizontal resistances, these losses were taken into account; but they were not taken into account in estimating the power required to overcome the vertical resistances. Hence, doubling the power previously given as useful effect, we have, in the illustration of the last number, for the power required to overcome the vertical resistances—

when $t = 60$ seconds,	162 Horse-power.
$t = 120$	"	130 "
$t = 180$	"	118 "

and for their total power—

when $t = 60$ seconds,	251 Horse-power.
$t = 120$	"	176 "
$t = 180$	"	153 "

Estimating this in the erroneous manner indicated at the close of the preceding article, we should have—

when $t = 60$ seconds,	$133 + 66 = 199$ Horse-powers
$t = 120$	"	$75 + 34 = 109$ "
$t = 180$	"	$59 + 22 = 81$ "

The per-centages of error of this method would be in this case 20, 38, and 47.

In order to contrast the result just obtained with another, take the same steamboat already described, and examine its ascent through a draw, constructed precisely the same as the last, but where the original velocity of five and a half miles an hour was increased to six. Take the time of ascent at 120 seconds. Five and a half miles per hour increased to six is 8 feet per second increased to 8.8. The corresponding hight of *remou* would be 0.23 feet. Here—

$$W = 1{,}875{,}000$$
$$b = 0.23$$
$$t = 120$$
$$v^{\circ} = (8 + 8.8) \div 2 = 8.4$$
$$l = 240$$

Which gives for the useful effect to be expended in overcoming the vertical resistances $P = W\,b\,[1 + (tv^{\circ} \div l)] = 2{,}242{,}500$ feet pounds. As this work is to be done in 120 seconds, we have $P = 34$ horse-power of useful effect, requiring on account of losses 68 horse-power of expenditure. The relative horizontal velocity of the boat is $v^{\circ} + (l \div t) = 10.4$ seconds, requiring for its accomplishment 104 horse-power. Hence the total power required will be $104 + 68 = 172$ horse-power. In the preceding illustration, where $t = 120$, the power required was 176 horse-power, showing that, of two draws constructed precisely alike, it requires less power to carry a steamboat up through one of them, where the velocity is six miles per hour than it does through the other, where the velocity is one mile less, or five miles per hour.

The question naturally arises: If the equivalent mean power required to carry a steamboat up through the draw exceeds the maximum power of the engines, will it be impossible for such a boat to make the ascent? By no means. When a moving body meets any resistance tending to check its motion, the inertia of the body comes into play as an active power to aid in overcoming the resistance. When the velocity of a steamboat is diminished by the resistances encountered in ascending a draw, then the inertia of every pound's weight of the boat and cargo comes to the aid of the engine. The measure of the power stored up in the inertia of a body is $(W \div 64.4)\,v^2$. W being its weight in pounds, and v its velocity in feet per second. It must be remembered, however, that this power is not available until the velocity begins to be checked. To apply this to the case last given:—What is the least power of the engine of the steamboat, to allow it to make the ascent, and leave the draw with an actual velocity of half a mile per hour?

Half a mile per hour is 0.7 feet per second, which, added to the velocity of the current above (8.) gives for the equivalent velocity with which the boat leaves the draw, 8.7 per second. Let x represent the number of horse-power required; y represent the greatest velocity which can be produced in still water by x; then y will be the relative velocity with which the boat approaches the draw. As the power required varies as the cube of the velocity, and as a velocity of 10 feet per second corresponds to 93 horse-power, we have $y^3 : 10^3 = x : 93$, and $x = (93 \div 1000)\,y^3$. The power (stored up in the inertia) with which the boat leaves the draw will be—$(1{,}875{,}000 \div 64.4) + 8.7^2 = 2{,}203{,}710$ feet lbs. The power consumed in making the ascent will be $172 \times 550 \times 120 = 11{,}352{,}000$ feet pounds. The power given out by the engine during the ascent will be $x \times 550 \times 120 = 66{,}000\,x$. The power (stored up in the inertia) with which the boat approaches the draw will be $(1{,}875{,}000 \div 64.4) \times y^2 = 29{,}115\,y^2$. The power present, *plus* the power developed during the ascent, must equal the power consumed during the ascent, *plus* the power remaining after the ascent. Hence we have—

$$29{,}115\,y^2 + 66{,}000\,x = 11{,}352{,}000 + 2{,}203{,}710 = 13{,}555{,}710.$$

$$29{,}115y^2 + 66{,}000 \times (93 \div 1000)y^3 = 13{,}555{,}710.$$

$$9{,}705\,y^2 + 2046y^3 = 4{,}518{,}570.$$

This equation may readily be solved by approximation; trying different values of y, until one sufficiently accurate is arrived at. As y must evidently be greater than the velocity (8.7), with which the boat leaves the draw, take at random $y = 10$.

When $y=10$	$9{,}705\,y^2 + 2046\,y^3$	$= 3{,}016{,}500$ (too small)
" $y=11$	" "	$= 3{,}897{,}531$ "
" $y=11.5$	" "	$= 4{,}395{,}190$ "
" $y=11.7$	" "	$= 4{,}605{,}420$ (too large)
" $y=11.6$	" "	$= 4{,}499{,}490$ (too small)

Hence as 11.6 is too small, and 11.7 too large, the true value lies between them; but, as we only carry the value of the velocity to one place of decimals, 11.6 is nearer the true value than 11.7, hence we have—

$$y = 11.6$$
$$x = (93 \div 1000)\,y^3 = 145.$$

Hence, instead of 172 horse-power, we have 145 horse-power, as the requisite capacity of the engine, to enable it to fulfill the given conditions.

In estimating the amount of power necessary to carry a steamboat up through a draw, we have considered only the power necessary to ascend the *remou*. The *remou* is situated just at the head of the piers, consequently before a boat reaches the foot of the *remou*, or commences its ascent, it will have to encounter *level* water moving with the velocity V. The resistance offered by this rapid current in the draw must be estimated according to the method previously given for estimating *horizontal* resistances, and the amount added to the power required for ascending the *remou*. It is evident that the greater the length of the draw, the longer the time the boat will be exposed to the rapid current V; consequently the draw should be made as short as possible.

In conclusion, I would state that the objects aimed at in the present investigation were twofold, first to obtain a simple and reliable method of measuring the increase of velocity and hight of *remou*, caused by the piers of bridges; and secondly, and principally, having obtained these values, to indicate a method by which the *obstruction to navigation* resulting from the accelerated velocity of the current, and the piling-up of the waters, might be accurately measured. That the method here suggested is faultless is not contended. I only claim that it is more accurate, full, and simple, than any other I have been able to find recorded.